Forman Journal of Economic Studies
Vol. 13, 2017 (January–December) pp. 1-32

Measurement of Income Inequality: A Survey

Muhammad Idrees and Eatzaz Ahmad[1]

Abstract

This paper reviews various inequality measures and finds that only few measures possess desirable properties of an inequality measure. Gini coefficient and the coefficient of variation are decomposable both in additive and non-additive forms and possess all the properties, except the stringent Pigou-Dalton condition given by Diminishing Transfers Axiom. Kakwani index and four popular generalized Gini indices posses all the properties but they are not decomposable additively or non-additively. Since sensitivity of an inequality measure to the location of income transfers also varies across various measures, hardly any measure can serve all the purposes and it is desirable to employ more than one measure in an empirical analysis of income inequality.

Keywords: Income Inequality, Measurement, Survey

JEL Classification: E01, E25, C80

1. Introduction

Measurement of inequality has been an area of great interest for statisticians and economists. Till the end of eighteenth century pure statistical measures like range and mean deviation were used to measure income inequality. However in the early nineteenth century a few specific measures of inequality were proposed. In 1905, Max Otto Lorenz proposed a revolutionary graphical measure of inequality known as Lorenz curve, from which Gini in 1912 derived a parametric measure of inequality, known as Gini coefficient. Since then a sizable literature on the measurement of income inequality has emerged. In another remarkable contribution in 1920 Dalton linked inequality to economic welfare and thereby originates the idea of normative inequality measures, which was further polished by Atkinson, (1970). Theil, (1967) derived inequality measure from the notion of entropy in information theory.

Later on it was realized that Gini coefficient and some other measures are characterized by certain rigidities. For example, Gini coefficient attaches more weight to income transfers affecting middle-income classes and not much weight

---

[1] The authors are respectively Associate Professor, School of Economics, Quaid-i-Azam University, Islamabad and Professor on SBP Memorial Chair, University of Peshawar, Peshawar. Corresponding author's email: midrees@qau.edu.pk

to income transfers within extreme income classes. In case of Lorenz curves the conclusion regarding the degree of inequality becomes ambiguous when the curves representing two different income distributions intersect each other. Moreover for comparing social welfare of two or more income distributions Lorenz curves are useful only when distributions have the same mean incomes. Such rigidities were seriously addressed and a number of generalized measures were proposed to overcome these inflexibilities during the 1980s. Among such generalizations basic focus of analysis was on the generalization of Lorenz curve and Gini coefficient. Shorrocks, (1983) introduced generalization of Lorenz curve, while a large number of generalizations of Gini coefficient were proposed, amongst which the ones proposed by Kakwani, (1980b), Donaldson and Weymark, (1980, 1983) and Yitzhaki, (1983) became popular. Shorrocks (1980) also proposed generalization of entropy indices.

Apart from measurement of inequality, another vital issue has been the splitting up of overall inequality into sub-components or sub-groups. There can be at least two ways to conduct decomposition of inequality, *i.e.* additive and non-additive. A measure is said to be additive decomposable when total inequality in the population under consideration can be broken into a weighted average of the inequalities existing between and within sub-groups of the population. In non-additive decomposition the focus of analysis is on the contribution of sub categories of the variable under consideration to total inequality. The literature shows that not all measures of inequality are decomposable. For example, generalized entropy indices and Ebert, (1999) indices are additive decomposable only, while Gini coefficient can be decomposed in both ways.

This study presents a comprehensive review of inequality measures. It consists of five sections. Section 2 proposes a classification of inequality measures into two groups that can be labeled as statistical measures and regular measures. The statistical measures of inequality include all the measures of dispersion that can also be used to measure inequality. The regular measures, on the other hand, include the measures that are meant for measuring inequality. These can further be sub-classified into four groups, namely ordinary measures, Lorenz curve and related measures; entropy measures and pure welfare based measures. Section 3 explains decomposing of inequality measure into sub groups and sources. Section 4 explains the desirable properties that a good inequality measure is supposed to possess and it evaluates all the measures considered on the basis of these properties. Finally, section 5 summarizes the entire discussion.

2. Classification of Inequality Measures

Measures of inequality can be classified in a number of ways. Sen, (1973) classified them into two categories namely positive and normative measures. Positive measures are those, which quantify the extent of inequality in an objective sense usually by employing statistical measures of dispersion. Normative measures are based on explicit formulation of social welfare function that indicates the welfare loss arising from an unequal distribution of income.[2] Thus positive measures seek to describe the existing pattern of income distribution as a single statistic without involving any value judgment, while normative measures base inequality on value judgments.

Positive measures include range, relative mean deviation, variance, coefficient of variation, Gini index, Lorenz curve, etc. Some well-known normative measures are Dalton measure and Atkinson index. As Sen, (1973) has pointed out, no firm line can be drawn between positive and normative measures and many of the positive measures are special cases of normative measures.

This paper proposes the classification of inequality measures into the groups of statistical measures of dispersion and regular measures of inequality. Statistical measures are designed to measure dispersion in any data and are also useable for measuring income inequality. These are positive measures of inequality and include, range, mean deviation, relative mean deviation, variance, coefficient of variation and variance of logarithms. The measures of inequality, purely meant for measuring inequality are hereby referred to as 'regular measures'. These measures can be further classified as ordinary measures, Lorenz curve and related measures, entropy measures and pure welfare based measures.

2.1. Statistical Measures

Some of the well-known statistical measures of dispersion are as follows.

2.1.1. Range

Range is defined as the difference between extreme values of a variable. While first time using range as a measure of inequality, Sen, (1973) divided it by arithmetic mean in order to make it a unit free measure. Denoting the income of

[2] Strictly speaking all the measures of inequality can be regarded as normative measures because first, in choosing any particular so-called positive measure one makes the value judgment that the chosen measure is a true indicator of inequality and second, many of the positive measures are special cases of certain normative measures.

income unit *i* by Y_i and arithmetic mean of income by $\overline{Y}$, the range as a measure of inequality is written as:

$$R = \left(MaxY_i - MinY_i\right)/\overline{Y} \quad (1)$$

The range takes values of zero and n for the extreme cases of perfect equality and perfect inequality respectively. Range is a simple measure of inequality but it completely ignores the distribution in between the extreme income levels.

2.1.2. Mean Deviation and Relative Mean Deviation

In order to consider the entire distribution, Bortkiewicz (1889) proposed mean deviation, defined below, which depends on the scale of measurement.

$$M = \sum_{i=1}^{n} \left|Y_i - \overline{Y}\right| \Big/ n \quad (2)$$

A related measure proposed by Turroni (1910), which is scale independent, is the relative mean deviation:

$$RM = \sum_{i=1}^{n} \left|Y_i - \overline{Y}\right| \Big/ \sum_{i=1}^{n} Yi = M/\overline{Y} \quad (3)$$

If income is distributed equally, the value of RM will be equal to zero, while in case of perfect inequality wherein one income unit holds the entire income, its value will be equal to $2(n-1)/n$. This measure (along with mean deviation) is not sensitive to income transfers between income units lying on any one side of mean income and it assigns equal weight to small as well as large deviations from the mean. Both these problems are overcome by variance and related measures.

2.1.3. Variance and Related Measures

Instead of ignoring signs, variance takes squares of mean deviations before averaging. It is given by

$$V = \sum_{i=1}^{n} \left(Y_i - \overline{Y}\right)^2 \Big/ n \quad (4)$$

A related measure is standard deviation, which is the square root of variance. Both these measures are scale dependent. This deficiency is overcome by coefficient of variation:

$$CV = \sqrt{V} \big/ \bar{Y} \tag{5}$$

In case of perfect equality the value of CV will be equal to zero and in case of perfect inequality its value will be equal to $\sqrt{n-1}$. A problem associated with this measure is that it is more sensitive to differences among rich or among poor income-units as compared to the differences among middle income-units.

Variance of the logarithm of income is another measure of inequality. Unlike variance, it is income scale independent and it makes inequality as sensitive to differences among rich or among poor income-units as to the differences among middle income-units. Denoting geometric mean of income by $\tilde{Y}$, the variance of log-income can be written as

$$V_L = \sum_{i=1}^{n} \left(\ln Y_i - \ln \tilde{Y} \right)^2 \Big/ n \tag{6}$$

Many studies (see Sen 1973) have used arithmetic mean in place of geometric mean in this formula. In any case a problem with this measures is that it become undefined when income of any unit equals zero.

2.2. Regular Measures

The regular measures of inequality meant purely for the measurement of inequality can be classified into four categories, which are discussed below.

2.2.1. Ordinary measures

Ordinary inequality measures can also be labeled as 'adhoc' inequality measures, as they are featured with numerous limitations.[3] We include Elteto and Frigyes indices in this category.

i. Elteto and Frigyes Indices

Elteto and Frigyes (1968) divided the entire population into two groups; those whose income is less than mean income and those whose income is equal to or greater than mean income and proposed the following three measures of inequality.

$$u = \bar{Y} \big/ \bar{Y}_L, \; v = \bar{Y}_G \big/ \bar{Y}_L, \; w = \bar{Y}_G \big/ \bar{Y} \tag{7}$$

[3] These measures do not fulfill many of the desirable properties of the inequality measures. For more details see Section 4 and Table 2.

where $\bar{Y}$ is the mean income in the entire population, $\bar{Y}_L$ the mean income of those whose income is less than $\bar{Y}$ and $\bar{Y}_G$ the mean income of those whose income is greater than or equal to $\bar{Y}$. The index *v* is an overall measure of inequality; while *u* and *w* indicate disparity of poor and rich income groups from the overall mean income. The three measures have the lower limit of one, while only *w* has an upper limit, which is n.

Elteto and Frigyes further proposed the following transformed measures.

$$u' = 1 - \frac{1}{u} = \frac{\bar{Y} - \bar{Y}_L}{\bar{Y}},\ v' = 1 - \frac{1}{v} = \frac{\bar{Y}_G - \bar{Y}_L}{\bar{Y}_G},\ w' = 1 - \frac{1}{w} = \frac{\bar{Y}_G - \bar{Y}}{\bar{Y}_G} \qquad (8)$$

The lower limit of each of these measures is zero. The upper limit of u' and v' is one, while that of w' is equal to $1 - 1/n$. Clearly $v = uw$, and $v' = u' + w' - u'w'$, that is only two of the three measures *u*, *v* and *w* or *u'*, *v'* and *w'* are mutually independent.

Kondor (1971) has shown that these three indices can be combined as follows to yield a value equal to one-half of the relative mean deviation, that is

$$\frac{(u-1)(w-1)}{(v-1)} = \left(\frac{p_L}{s_L} - 1\right)\left(\frac{1-s_L}{1-p_L} - 1\right) \Big/ \left(\frac{p_L}{s_L}\frac{1-s_L}{1-p_L} - 1\right) = p_L - s_L = \frac{RM}{2} \qquad (9)$$

where p_L and s_L are respectively the population and income shares of the poor group of population.[4]

ii. Lorenz Curve and Related Measures

Lorenz curve is one of the most widely used tools to describe state of inequality. A large numbers of inequality measures are directly based on Lorenz curve. The most common among them are Gini coefficient, Schultz index and Kakwani index.[5]

---

[4] Schultz's (1951) measured, discussed in the sub-section 2.2.2, is also equal to half of relative mean deviation.

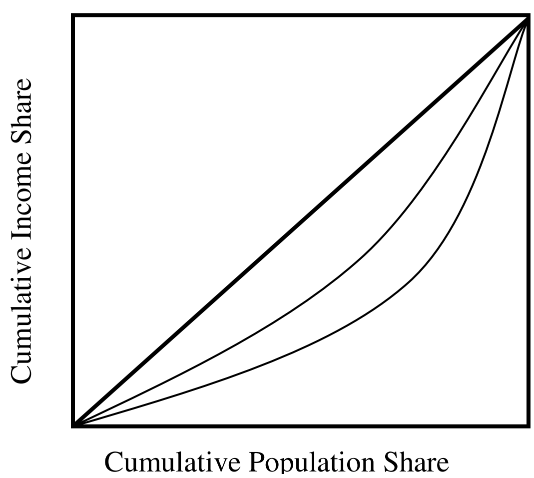
[5] Schultz Coefficient is simply equal to one half of the relative mean deviation' so the later can also be based on Lorenz curve. However, since relative mean deviation has been derived independently, it will not be discussed here. Similarly the measures proposed by Elteto & Frigyes can also be related to relative mean deviation. But again these measures will not be discussed here, as they have not been derived from Lorenz curve.

Lorenz curve, named after a US statistician Max Otto Lorenz and introduced in 1905 is the relationship between the cumulative percentages of incomes (placed in ascending order) measured along the vertical axis and the corresponding cumulative percentages of income units measured along the horizontal axis.[6] Let incomes be denoted by $Y_1,...,Y_n$ such that $0 < Y_1 \leq Y_2,... \leq Y_n$ and the corresponding income shares by $s_1,...,s_n$. Then cumulative shares of income units and incomes are i/n and $q_n = \sum_{i=1}^{n} s_i$ respectively. The Lorenz curve can now be constructed by taking combinations of i/n and q_i.

$$(0,\ 0), \left(\frac{1}{n},\ q_1\right), \left(\frac{2}{n},\ q_2\right), \ldots, \left(\frac{n-2}{n},\ q_{n-2}\right), \left(\frac{n-1}{n},\ q_{n-1}\right), (1,\ 1) \tag{10}$$

The Lorenz curve, as plotted in Figure 1, shows that the curve closer to the line of perfect equality (the diagonal line) represents a more equal distribution of income as compared to the one that is relatively away from the line of perfect equality.

Figure 1: Lorenz Curve



Although Lorenz curve is simple and a widely accepted measure of inequality, yet it is not free from limitations. For example, conclusion regarding the degree of inequality becomes ambiguous when the curves representing two different income

[6] Kuan Xu (2003) has pointed out that the concept of Lorenz curve was initially hinted by Sir Leo Chiozza Money (1905) but it was formally proposed by Lorenz.

distributions intersect each other. Furthermore the Lorenz curve does not provide a numeric measure of inequality.

Atkinson (1970) explained the ethical strength of Lorenz curve by relating it to social welfare. In its modified form, the Lorenz curve is related to the basic characteristics of social welfare function. This is best explained in Atkinson's Theorem, called Lorenz Dominance Criterion, stated as follows.

Theorem I: This theorem states that for two income distributions A and B, having identical means, social welfare in distribution A is greater than social welfare in distribution B if Lorenz curve of distribution A lies everywhere above the Lorenz curve of distribution B, provided that the underlying social welfare function is individualistic, non-decreasing, symmetric, additive and strictly concave.

Although Lorenz Dominance is a useful and important theorem, but as a criterion of welfare comparison it has two limitations; it permits comparison only when distributions have same mean incomes and it does not provide comparison between intersecting Lorenz curves.

iii. Generalized Lorenz Curve

This curve was proposed by Shorrocks (1983) to overcome the limitations of Lorenz Dominance criteria to some extent. It is obtained by scaling up Lorenz curve by mean income. It is obtained by plotting the combinations of cumulative population shares i/n and cumulative income shares multiplied by mean income $q_i\bar{Y}$, that is,

$$(0,\ 0),\left(\frac{1}{n},\ q_1\bar{Y}\right),\left(\frac{2}{n},\ q_2\bar{Y}\right),\ldots,\left(\frac{n-2}{n},\ q_{n-2}\bar{Y}\right),\left(\frac{n-1}{n},\ q_{n-1}\bar{Y}\right),\left(1,\ \bar{Y}\right) \qquad (11)$$

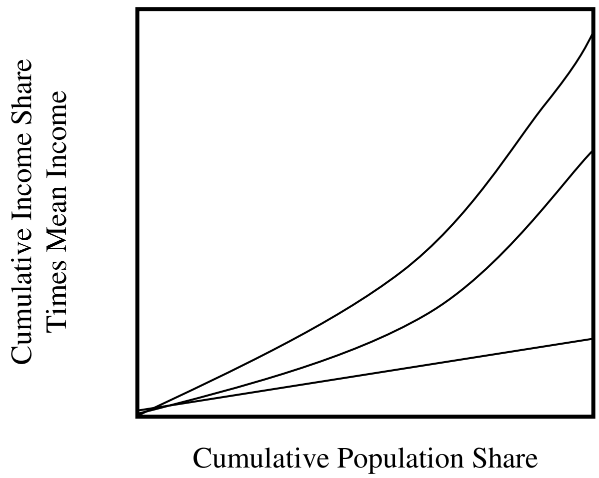
Height of the point where the generalized Lorenz curve terminates shows the mean income and convexity measures the extent of inequality. As with the ordinary Lorenz curve, the higher the degree of convexity, the higher will be the extent of inequality and vice-versa. As an example three generalized Lorenz curves are shown in Figure 2. The lowest curve represents equal distribution, while the upper-most curve indicates the maximum degree of inequality.

iv. Social Welfare and Generalized Lorenz Curve

Shorrocks (1983) has proposed Generalized Lorenz Dominance criterion by which welfare comparison can be made even between the distributions having different mean income and/or intersecting ordinary Lorenz curves.

Theorem 2: For two income distributions A and B, social welfare in the distribution A is greater than social welfare in the distribution B if the generalized Lorenz curve of distribution A lies everywhere above the generalized Lorenz curve of distribution B, provided that underlying social welfare function is individualistic, non decreasing, symmetric, additive and strictly concave.

Figure 2: Generalized Lorenz Curve



Note, however, that even Generalized Lorenz Dominance criterion' fails if the generalized Lorenz curves intersect each other. Thus both Lorenz Dominance and Generalized Lorenz Dominance criteria provide incomplete ranking of welfare states. We now consider the parametric measures of inequality that can be derived from Lorenz curve.

v. Kakwani Index

Kakwani, (1980a) introduced the following measure of inequality based on Lorenz curve.

$$K_1 = \left(l - \sqrt{2}\right)/\left(2 - \sqrt{2}\right) \qquad (12)$$

where 'l' is the length of Lorenz curve. If each income unit receives the same income, the length will be equal to $\sqrt{2}$ and if one income unit holds the entire income, the length will be equal to 2. Thus the value of Kakwani index lies between zero and one.[7]

vi. Schultz Index

Schultz, (1951) proposed another measure of inequality based on Lorenz curve. It is defined as the value of the maximum discrepancy (measured by horizontal distance) between the line of perfect equality and Lorenz curve. It is given by:

$$S = \frac{1}{2n\overline{Y}} \sum_{i-1}^{n} \left| Y_i - \overline{Y} \right| \tag{13}$$

Schultz coefficient measures the proportion of total income that would have to be transferred from those whose income is above mean income to those whose income is below mean in order to attain perfect equality. That is why it is also known as 'maximum equalization percentage' or 'Robin Hood index'. Schultz coefficient is equal to one half of relative mean deviation, so it shares all the merits and demerits of relative mean deviation.

vii. Gini Coefficient

Gini coefficient, attributed to Gini, (1912), is by far the most popular measures of income inequality.[8] There are at least three approaches to define Gini coefficient. The first one, called geometric approach, expresses Gini coefficient as the ratio of area between the line of absolute equality and the Lorenz curve to the total area below the line of absolute equality. Rao, (1969) has given following formula to calculate Gini coefficient through geometric approach:

$$G = \sum_{i=1}^{n-1} \left(P_i q_{i+1} - P_{i+1} q_i \right) \tag{14}$$

where P_i is the cumulative population share and q_i is the cumulative income share of the income unit i, when all income units are arranged in ascending order of income.

---

[7] Gini coefficient attaches more weight to transfers of income near the mode of the distribution than in any one of the tails, while Kakwani index attaches more weight to transfers at the lower end than at the middle and upper ends of distribution (See Kakwani 1980a). Discussion on Gini coefficient is given later in this section.

[8] David (1968) has pointed out that Gini Coefficient, as given by relative mean difference, was developed much earlier by F. R. Helmert in 1870s. However, its link with Lorenz curve was established by Gini himself.

The second approach is attributed to Gini himself who referred to the Gini coefficient as 'concentration ratio'. In the words of Gini, (1921) "the concentration ratio is the quotient of the mean difference by the twice the arithmetic mean". Denoting incomes of the income units i and j by Y_i and Y_j, the mean income by $\overline{Y}$ and the number of income units by n, Gini coefficient according to this approach can be written as (see Kendall and Stuart 1963):

$$G = \frac{1}{2n^2\overline{Y}} \sum_{i-1}^{n} \sum_{j-1}^{n} \left| Y_i - Y_j \right| \tag{15}$$

The third approach expresses Gini coefficient as a function of covariance between incomes and their ranks and it is given by (see Anand 1983):[9]

$$G = 2\,Cov\left(i, Y_i\right) \big/ n\overline{Y} \tag{16}$$

Gini coefficient lies between zero and one; zero representing perfect equality and one perfect inequality. It provides a meaningful interpretation of Lorenz curve. Moreover it is based on more direct approach and does not take arbitrary squares, as in case of variance and related measures. However, a problem with Gini coefficient is that it attaches more weight to income transfers affecting middle-income classes and not much weight to income transfers within extreme income classes. This problem is somewhat solved by Generalized Gini indices.

viii. Generalized Gini Indices

Generalized Gini indices are normative in nature, as these can be made more or less sensitive to income transfers at any part of income distribution. Generalized Gini indices are based on weighted gaps between the line of perfect equality and Lorenz curve along different locations of a given income distribution and various generalizations differs in their weighting scheme. This property is often referred to as ethical flexibility.

Kakwani, (1980b) introduced the following generalization, in which the parameter α represents sensitivity of the inequality index to income transfers between different income units.

---

[9] Gini coefficient can also be calculated by many other ways. A good description is available in Anand (1983).

$$G_{(\alpha)} = \left[(n-1) \Big/ n \left(\sum_{i=1}^{n} i^{\alpha} - n \right) \right] \frac{1}{\overline{Y}} \sum_{i=1}^{n} \left(\overline{Y} - Yi \right) (n + 1 - i)^{\alpha} \qquad (17)$$

For $\alpha > 1$ ($\alpha < 1$) more weight is given to transfers in lower (upper) tail of income distribution. It coincides with ordinary Gini coefficient when $\alpha = 1$. Donaldson and Weymark, (1980) and Yitzhaki, (1983) have also proposed generalization of Gini coefficient, but these are cardinally equivalent to Kakwani's generalized Gini index (see Chakrawarty, 1988 and Yitzhaki, 1983).

More recently Chotikapanich and Griffiths (2001) proposed the following generalization, where s_i, p_i and P_i are respectively income share, population share and cumulative population share of the ith income unit and υ is an inequality aversion parameter:

$$G_{(v)} = 1 + \sum_{i=1}^{n} \left[\frac{s_i}{p_i} \right] \left[\left(1 - P_i\right)^{v} - \left(1 - P_{i-1}\right)^{v} \right], \; v > 1 \qquad (18)$$

For $v < 2$ ($v > 2$) more weight is given to transfers in upper (lower) tail of the income distribution. The index coincides with ordinary Gini coefficient when $v = 2$.

2.2.2. Entropy Measures

Another popular class of inequality measures is known as entropy measure, which is derived from the notion of 'entropy' in information theory. It includes two types of measures namely Theil entropy measures and generalized entropy indices. The basic idea behind entropy is that events that differ from what was expected, should receive more weight than the events that confirm with prior expectations.

i. Theil's Entropy Measures

If s is the probability that a certain event will occur, the information content $h(s)$ of noticing that the event has occurred must be a decreasing function of s. One possible way to express such a function is in the form of logarithm of reciprocals, that is $h(s) = \ln(1/s)$. With n possible events with probabilities $s_1, \ldots, s_n$, the entropy can be defined as sum of the information contents of all the events weighted by their respective probabilities: $H(s) = \sum_{i=1}^{n} s_i h(s_i) = \sum_{i=1}^{n} s_i \ln(1/s_i)$

The information content is zero when one of the events has probability equal to one; that is one draws no information from the occurrence of an event that was anticipated with certainty. The information content is at its maximum when $s_i = \frac{1}{n}$ and, hence $H = \ln(n)$.

If s_i is interpreted as the income share of the income unit *i*, $H(s)$ will look like a measure of equality. Thus subtracting entropy $H(s)$ from its maximum value $\ln(n)$, the latter representing perfect equality, yields an index of inequality, known as Theil's first entropy index of inequality: $T_1 = \ln(n) - H(s) = \sum_{i=1}^{n} s_i \ln\left[s_i/(1/n)\right]$. Since $s_i = Y_i / n\bar{Y}$, we can further write Theil's index as:

$$T_1 = \frac{1}{n}\sum_{i=1}^{n}\left(Yi/\bar{Y}\right)\ln\left(Yi/\bar{Y}\right) \tag{19}$$

Theil, (1967) has interpreted T_1 as "the expected information of a message that transforms population shares into income shares". In case of perfect equality the income share of each income unit is equal to the corresponding population share and hence the index takes the value equal to zero. On the other hand, if income share of one income unit is equal to one and that of everyone else is equal to zero then T_1 assumes the value equal to $\ln(n)$. Furthermore, higher the difference between income shares and population shares, the higher will be the value of Theil index. Considering this principle, Anand (1983) restated Theil index as a general distance function that measures divergence between income and population shares. Although Theil index is frequently used for measuring income inequality, Sen (1973) opined that it lacks intuitive sense and is just an arbitrary formula.

Theil's second measure is obtained by interchanging the roles of population and income shares in the formula $T_1 = \sum_{i=1}^{n} s_i \ln\left[s_i/(1/n)\right]$ to yield $T_2 = \sum_{i=1}^{n}(1/n)\ln\left[(1/n)/s_i\right] = \frac{1}{n}\sum_{i=1}^{n}\ln\left[(1/n)/s_i\right]$. Since $s_i = Y_i / n\bar{Y}$, Theil's second measure can be written as

$$T_2 = \frac{1}{n}\sum_{i=1}^{n}\left[\ln\left(\overline{Y}/Y_i\right)\right] \tag{20}$$

It is apparent from the above that Theil's second measure of inequality is equal to mean log deviation or log of the ratio of arithmetic mean to geometric mean of income. The lower limit of Theil's second measure is zero but it has no upper limit. In actual practice both the measures of Theil do not consider zero incomes, as log of zero is undetermined.

ii. Generalized Entropy Indices

Shorrocks, (1980) presented the following class of generalized entropy indices.

$$I_c = \frac{1}{n}\frac{1}{c(c-1)}\sum_{i=1}^{n}\left[\left(Yi/\overline{Y}\right)^c - 1\right], \quad c \neq 0, c \neq 1 \tag{21a}$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left(Yi/\overline{Y}\right)\ln\left(Yi/\overline{Y}\right) = T_1, \quad c = 1 \tag{21b}$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left[\ln\left(\overline{Y}/Y_i\right)\right] = T_2, \quad c = 0 \tag{21c}$$

Note that for $c = 2$ the index becomes one half of the squared coefficient of variation and cardinally equivalent to Herfindal index, which is a measure of industrial concentration. As the value of c increases, the measure becomes more sensitive to changes in the upper tail of the income distribution. The lower bound of I_c is zero, while the upper bound varies with c. With $c = 0$ the measure has no upper limit, while with $c = 1$ its upper limit is $\ln(n)$. If $c > 0$ and $c \neq 1$ and all incomes are positive then the upper bond of I_c will be $\left(n^{c-1} - 1\right)/c(c-1)$.

2.2.3. Pure Welfare Based Measures

The measures of inequality discussed so far are positive measures in their specific original forms. The generalized forms of these measures base inequality on value judgment about the sensitivity parameter. In this sense the generalized measures become normative in nature. Now we describe the class of inequality measures, which are not generalized forms of positive measures but are pure normative measures.

i. Dalton's Measure

Dalton, (1920) was the first to introduce the idea that inequality measurement should relate to economic welfare. His measure is based on utilitarian framework and it uses the intuition that income inequality results in loss of social welfare. As shown in Kakwani (1980a), Dalton's measure can be written as

$$D = 1 - \sum_{i-1}^{n} U(Y_i) \Big/ nU(\bar{Y}) \tag{22}$$

With perfect equality $\sum_{i=1}^{n} U(Y_i) = nU(\bar{Y})$, hence $D = 0$. Assuming diminishing marginal utility if incomes are unequally distributed, we'll have $nU(\bar{Y}) > \sum_{i=1}^{n} U(Y_i) > 0$ and, hence, $0 < D < 1$. It follows that greater the difference between $\sum_{i=1}^{n} U(Y_i)$ and $nU(\bar{Y})$, the greater will be the value of D and higher will be the degree of inequality.

Dalton's measure provides a general rule for defining inequality in terms of welfare. For actual measurement of inequality the utility function needs to be parameterized. Using an inequality aversion parameter ε, Cowell (2000) redefined Dalton's index as follows.

$$D = 1 - \frac{\frac{1}{n}\sum_{i=1}^{n}\left[Y_i^{1-\varepsilon} - 1\right]}{\bar{Y}^{1-\varepsilon} - 1}, \quad \varepsilon > 0 \tag{23}$$

The condition $\varepsilon > 0$ implies social preference for equality. The larger the value of ε, the greater will be the weight attached to transfers at the lower end of the distribution. As $\varepsilon \to \infty$, the welfare function becomes Rawalsian, i.e., welfare depends on income of the poorest member of society. On the other hand as $\varepsilon \to 0$, the welfare function becomes linear in income and, hence, invariant to redistribution of income. Cowell has pointed out the limitation of Dalton's index that its value does not necessarily increase with ε, the presumed inequality aversion parameter.

ii. Atkinson's Measure

Atkinson, (1970) criticized Dalton's index on the grounds that it is variant with respect to positive linear transformations of utility function. Atkinson suggested an alternative measure based on the concept of equally distributed equivalent income Y_e, which if equally distributed will make the welfare level exactly equal to the level generated by actual distribution of the given aggregate income. That is $Y_e = Y \left| nU(Y_e) = \sum_{i-1}^{n} U(Yi) \right.$. If the function $U(Y)$ is concave, Y_e cannot be larger than the mean income $\overline{Y}$. The difference between these two can be interpreted as the welfare loss due to inequality. Thus greater is the divergence between Y_e and $\overline{Y}$, the greater will be the level of inequality and vice versa. Atkinson's measure can be obtained by dividing the difference between $\overline{Y}$ and Y_e by $\overline{Y}$, that is:[10]

$$A = \left(\overline{Y} - Y_e\right) \Big/ \overline{Y} = 1 - Y_e \Big/ \overline{Y} \tag{24}$$

The specific form of Atkinson's index is given by:

$$A_{(\varepsilon)} = 1 - \frac{1}{\overline{Y}} \left[\frac{1}{n} \sum_{i=1}^{n} (Yi)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}, \quad \varepsilon > 0, \varepsilon \neq 1 \tag{25a}$$

$$A_{(\varepsilon)} = 1 - \frac{1}{\overline{Y}} \prod_{i=1}^{n} (Yi)^{1/n}, \quad \varepsilon = 1, \tag{25b}$$

If incomes are equally distributed and hence $Y_e = \overline{Y}$ then the value of Atkinson's index will be equal to zero. If all income is given to just one income unit, Y_e will approach to zero and Atkinson's index will take the value equal to one. In general when incomes are unequally distributed we shall have $0 < A < 1$. Ebert (1999) has interpreted 'A' as the fraction of mean income that is lost per income unit due to inequality.

Atkinson's index also has certain limitations. For example, its values are not comparable across societies even for a given value of the inequality aversion parameter ε because one cannot claim that all societies have the same attitude towards inequality. However, this argument can equally be applied to other inequality measures in general and the ones involving inequality aversion parameters in particular.

---

[10] Sen (1972) pointed out that Atkinson's measure requires that the function U(Y) be concave but not strictly concave,. i.e. $U' > 0$ *and* $U'' \leq 0$.

iii. Ebert's Measure

Ebert (1999) introduced the following measure based on the concept of equally distributed equivalent income, which Ebert interpreted as representative income or average standard of living evaluated by the underlying social welfare ordering.

$$E = \left(\overline{Y} - Y_e\right)/Y_e = \overline{Y}/Y_e - 1 \qquad (26)$$

In specific terms Ebert (1999) proposed following formula for the measurement of inequality.

$$E_{(\varepsilon)} = \frac{\overline{Y}}{\left[\frac{1}{n}\sum_{i=1}^{n}(Yi)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}} - 1, \quad \varepsilon > 0, \varepsilon \neq 1 \qquad (27a)$$

$$E_{(\varepsilon)} = \frac{\overline{Y}}{\prod_{i=1}^{n}(Yi)^{1/n}} - 1, \quad \varepsilon = 1 \qquad (27b)$$

Ebert's index has the lower limit equal to zero and has no upper limit. Note that while Atkinson's index measures welfare loss due to inequality as a proportion to mean income, Ebert's index expresses the welfare loss as a proportion to equally distributed equivalent income. Obviously Ebert's index and Atkinson's index are ordinally equivalent because $E_{(\varepsilon)} = A_{(\varepsilon)}/\left[1 - A_{(\varepsilon)}\right]$. Ebert (1999) has further pointed out that every Ebert's index is also ordinally equivalent to the corresponding generalized entropy measure for $1-\varepsilon = c < 1$. This concludes the description of the measurement of inequality. The next section is focused on decomposition analysis of the inequality measures.

3. Decomposition of Inequality Measures

An important task in analyzing inequality is to work out its structure and sources. For example, it is worthwhile to know how the income inequality in a country is accounted for by inequality within its different regions and the inequality between the regions. Similarly it is worthwhile to express inequality into different sources of income, such as wages, rents, etc. Inequality decomposition is a standard technique for examining the contribution of sub-groups of population, income sources/types and characteristics of income units to the overall inequality. Decomposition analysis is helpful in pointing out the sources and incidence of inequality.

There can be at least two ways to conduct decomposition of inequality, *i.e*. either by splitting up of the population, called sub-group decomposition or by division of income (or other such variables), known as source-decomposition. It is the objective of decomposition, which determines what type of decomposition is to be carried out. The sub-group and source decomposition can appropriately be distinguished as additive and non-additive decomposition. A measure is said to be additive decomposable when total inequality of population can be broken down into a weighted average of the inequalities existing between and within sub-groups of populations. In non-additive decomposition the focus of analysis is on the contribution of sub-categories of income (or other such variables) to total inequality, rather than on how total inequality is sub-divided between and within sub-groups. Further detail on the two types of decomposition techniques are discussed as follows.

3.1. Additive Decomposition

An additive decomposable measure is the one that can split total inequality (I_T) into a weighted average of the inequality existing within sub-groups of the populations (I_W) and the inequality existing between the sub-groups (I_B). If g_1 $,\ldots, g_K$ are the K sub-groups of population then an additive decomposable measure can be written as follows.

$$I_T(g_1,\ldots,g_K) = [I_W(g_1) + \ldots + I_W(g_K)] + [I_B(g_1,\ldots,g_K)] \tag{28}$$

where $I(g_1,\ldots,g_K)$, $I_W(g_k)$ and $I_B(g_1,\ldots,g_K)$ denote inequality in the entire population, within sub-group k and between sub-groups 1 to K respectively. The between-group component can be defined as the level of inequality when incomes within each sub-group have been equalized, i.e., each income unit within a sub-group is given the mean income of the sub-group.[11] Likewise the within sub-group component can be defined as the value of the inequality index when all the between group income differences are suppressed. In order to eliminate the between group income differences mean incomes across the sub-groups are equalized to the overall mean income through equi-proportionate changes in incomes of the individual income units.

[11] There are two views regarding the equalization of all within group incomes. The traditional view followed by Shorrocks (1980, 1984) and Cowell (1980) is to assume that each income unit receives the mean income of its sub-group. Ebert (1999) has, however, used equally distributed equivalent income for this equalization.

Anand (1983) has pointed out that Theil's second measure and variance of log-incomes are the only two measures which are additive decomposable under this strict definition. Cowell has further shown that in practice the variance of log-incomes is not properly decomposable because of the complexity in disentangling the within-group and between-group inequality components. A more relaxed definition of the within group component, which has broader application, is that it is the weighted sum of inequality indices of all the sub-groups. In the following analysis we shall consider this relaxed definition in order to add more inequality measures in the class of additive decomposable measures, which along with decomposability also need to satisfy other desirable properties of an inequality measure.

Shorrocks (1980) has shown that any inequality measure that satisfies diminishing-transfer axiom, principle of population, income-scale independence and decomposability must belong to generalized entropy class or its ordinal transformations.[12] Shorrocks has, however, pointed out that with the exception of Theil's two measures, no entropy index satisfies adding-up condition, i.e., within-group component weights do not sum to one and the decomposition coefficients are dependent on the between groups contribution. A related problem lies in interpreting contributions of the two components to total inequality: In words of Shorrocks "Interpretation (i) suggests a comparison of total inequality with the value which would arise if inequality was zero within each age group, but the difference in mean income between age groups remained the same. For the additive decomposable indices this would eliminate the total within group-term and leave only the between group contribution. Interpretation (ii) suggests a comparison of total inequality with the value, which would result if the mean incomes of the age groups were made identical, but inequality within each group remained unchanged. This eliminates the between group term in the decomposition equation; but the reduction in inequality is not simply between groups inequality, because in general, changing the age-group means will also affect the decomposition coefficients and hence the total within group contribution". Shorrocks further pointed out that the two interpretations are reconcilable if and only if the weights assigned to income units are independent of income shares. Only Theil's second measure (also called mean log deviation), in which weights are population share, satisfies this requirement. Hence Theil's second measure is the most satisfactory additive decomposable measure among the class of generalized entropy indices.

[12] These are the desirable properties of inequality measures, which will be explained in section 4.

Foster and Shneyerov (2000) have explored 'path independent decomposition' property for neat decomposition in which within group and between groups components of total inequality are mutually independent. They conclude that mean log deviation or Theil's second measure has a path independent decomposition when deviations are taken from arithmetic mean, and variance of logarithms is path independent when deviations are taken from geometric mean.

Gini coefficient is another additive decomposable inequality measure. However, it's neat additive decomposition remained an unsettled issue for a number of years. Bhatacharya and Mahalanonis (1967) were first to work on the sub-group decomposition of Gini coefficient and they ended up with an additional cross term. Paytt (1976) and Das and Parikh (1982) also found the same results. Mookherjee and Shorrocks (1982) arrived at the conclusion that there is no meaningful interpretation of this cross term. Shorrocks (1984) showed that neat additive decomposition of Gini coefficient, without cross term is possible if incomes of all income units in one sub-group are less than those in the other sub-group. Silber (1989), Yitzhaki and Lerman (1991) and Yitzhaki (1994), however, suggested different interpretations of cross term (see Kuan 2003). Dagum (1997) has shown that Gini coefficient is neatly decomposable without cross term if sub-groups of populations do not overlap and if they do overlap then Gini coefficient can be additively decomposed into three components; inequality within sub-groups of population, the net contribution of extended Gini inequality between the sub-groups and the contribution of the intensity of trans-variation (overlapping effect) between sub groups of population. The word trans-variation stands to the fact that the differences in incomes across sub-groups considered are of opposite sign than the difference in mean incomes of the corresponding sub-groups.

More recently Ebert (1999) has presented a new family of additively decomposable measures. Ebert's measures, generalized entropy measures and Atkinson's indices are ordinally equivalent, so they provide the same ranking of inequality across a set of populations. Atkinson's indices are not additively decomposable; while Ebert's and generalized entropy indices are decomposable.[13] The selection between Ebert's measures and generalized entropy indices for decomposition depends upon objective of decomposition. In words of Ebert (1999) "if the focus is on income (describing opportunities) generalized entropy measures seem to be more suitable; if the distribution of living standard is

[13] de la Vega and Urrutia (2003) have, however, presented factorial decomposition of Atkinson's indices in which total inequality can be written as product of within group and between groups inequality.

relevant Ebert's indices should be preferred." The additive decomposition of each additive decomposable measure is given in Table: A1 (Appendix – A).

3.2. Non-Additive Decomposition

Non-additive decomposition focuses on the contribution of components of the variable analyzed, such as income or consumption, to total inequality. Shorrocks (1982) has shown that Gini coefficient, variance and coefficient of variation are the only well-known measures of inequality that can be decomposed by this criterion. The decomposition of Gini coefficient is straightforward. Denoting the cumulative shares of income component k by q_i^k, the concentration coefficient for the income component k, which is like Gini coefficient given by (15) but after placing the income component in the ascending order of income, can be calculated as

$$C_k = \sum_{i=1}^{n-1} \left(P_i q_{i+1}^k - P_{i+1} q_i^k \right) \tag{29}$$

It is straightforward to show that Gini coefficient of income is equal to the weighted sum of the concentration coefficients of income components, where weights are the shares of aggregate income components in the aggregate income, that is,

$$G = \sum_{k=1}^{K} \left[s_k \left(C_k \right) \right] \tag{30}$$

The contribution of income component k to total inequality (denoted by O_k^G) can be obtained as:

$$O_k^G = s_k \left(\frac{C_k}{G} \right) \tag{31}$$

This effect will be positive, zero and negative when $C_k > G$, $C_k = G$ and $C_k < G$ respectively.

Next, the variance $V(Y)$ and the squared coefficient of variation $\left[CV(Y) \right]^2$ can be decomposed, following Shorrocks (1982), as

$$V(Y) = \sum_k Cov(Y_k, Y) = \sum_k V(Y_k) + \sum_k \sum_{j \neq k} \rho_{kj} \left[V(Y_k) \right]^{1/2} \left[V(Y_j) \right]^{1/2} \tag{32}$$

$$[CV(Y)]^2 = \sum_k \frac{Cov(Y_k, Y)}{\bar{Y}^2} \qquad = \sum_k \frac{V(Y_K)}{\bar{Y}^2} + \sum_k \frac{\sum_{j \neq k} \rho_{kj} [V(Y_k)]^{1/2} [V(Y_j)]^{1/2}}{\bar{Y}^2}$$

$$= \sum_k \frac{\bar{Y}_k^2}{\bar{Y}^2} \frac{Cov(Y_k, Y)}{\bar{Y}_k^2} = \sum_k \frac{\bar{Y}_k^2}{\bar{Y}^2} [CV(Y_k)]^2 + \sum_k \frac{\bar{Y}_k}{\bar{Y}} \sum_{j \neq k} \frac{\bar{Y}_j}{\bar{Y}} \rho_{kj} [CV(Y_k)][CV(Y_j)] \tag{33}$$

where ρ_{jk} is the correlation coefficient between income components j and k. It is easy to verify that the contribution of each income component to the overall inequality for variance and the square of coefficient of variation is the same and is given by:.

$$O_k^V = O_k^C = \frac{Cov(Y_k, Y)}{V(Y)} = \rho_k \sqrt{\frac{V(Y_k)}{V(Y)}} = \frac{V(Y_k)}{V(Y)} \left[1 + \sum_{j \neq k} \rho_{kj} \sqrt{\frac{V(Y_j)}{V(Y_k)}} \right] \tag{34}$$

where ρ_k is the correlation coefficient between Y_k and Y. This concludes our discussion on the decomposition of inequality measure.

4. Desirable Properties of an Inequality Measure

Each inequality measure has certain qualities of its own. One way of selecting a desirable inequality measure is to adopt 'axiomatic approach', according to which an ideal measure should possess certain characteristics. A summary of these properties, mostly based on Litchfield (1999), is given as follows.

4.1. The Pigou-Dalton Transfer Principle

The inequality measure should indicate increase (decrease) in inequality as a result of regressive (progressive) transfers of income. That is an income transfer from a poor (rich) to a richer (poorer) income unit should increase (decrease) the value of the inequality measure or at least leave it unchanged, provided their ranks do not change.

A stronger version of this axiom is 'Diminishing Transfers Axiom*' (DTA)*, according to which if equal amounts of incomes are taken from two income units with incomes Y_i and Y_j, where $Y_i < Y_j$ and given to income units with incomes $Y_i - c$ and $Y_j - c$, where $c > 0$, such that income transfers are rank preserving, then income transfer from Y_i will reduce inequality by a greater extent as compared to the reduction caused by income transfer from Y_j

4.2. Principle of Population

An inequality measure should be invariant to replication of population. Thus merging two or more identical distributions should not alter the degree of inequality. This axiom indicates that the extent of measured inequality should not depend on size of the population.

4.3. Symmetry

Inequality measure should be independent of any characteristics of income units other than their incomes or other welfare indicators being measured.

4.4. Income Scale Independence

Inequality measure should be invariant to uniform proportional changes in incomes. Fields and Fei (1978) have shown that an index that satisfies the above four properties (excluding the stronger Diminishing Transfer axiom) will also fulfill Lorenz Dominance criterion. Such a measure is also referred to as Lorenz Consistent measure.

4.5. Principle of Addition

If a positive (negative) constant is added to the incomes of the all income units, the value of inequality measure should indicate decrease (increase) in the degree of inequality. The basic idea is that if unequally distributed incomes are supplemented with equally distributed transfers, the incomes of the poor will rise relative to the incomes of rich, thereby reducing the degree of inequality. It is easy to verify that Income Scale Independence and Pigou-Dalton Transfer Principle are sufficient, though not necessary, for the fulfillment of the Principle of Addition axiom.

4.6. Decomposability

An inequality measure should be decomposable, both additively and non-additively.

4.7. Defined Limits

An inequality measure should have defined and interpretable limits independent of the size of population. In most cases the lower limit of an inequality measure is zero, showing perfect equality and upper limit is one, showing perfect inequality. This property allows interpretable assessment of the degree of inequality and its comparison across populations.

Not all measures of inequality satisfy the above properties. Table 1 shows that the principle of population and symmetry is the only criteria that satisfy all measures. Furthermore with the exception of mean deviation, variance and Dalton measure, all the measures are also income scale independent. As far as Pigou-Dalton Transfer condition is concerned range satisfies this condition if and only if the recipient is the poorest income unit and/or the donor is the richest income unit, as it takes into account only two extreme incomes. Mean deviation, relative mean deviation and the measures proposed by Elteto and Frigyes and Shultz index satisfy this condition if income of the recipient is below mean income and income of the donor is above mean income, that is, these measures are not sensitive to transfers between income units lying on one side of the mean income. Anand (1983) has shown that variance of log-incomes also does not satisfy Pigou-Dalton condition for transfers within incomes above $e\tilde{Y}$, where e is the base of natural logarithm and $\tilde{Y}$ is geometric mean of incomes. All other measures satisfy this property. Variance, coefficient of variation, Gini coefficient and generalized Gini coefficients that satisfy the basic Pigou-Dalton transfer condition, fail to meet conditions for the stronger version, that is, Diminishing Transfer axiom. Only one of the generalizations of Gini coefficient presented by Chotikapanich and Griffiths satisfies the diminishing transfer axiom and that too for the sensitivity parameter $v > 2$.

It follows from the discussion so far that most of the well kwon measures, specifically coefficient of variation, Kakwani index, the Gini and Entropy classes, Atkinson index and Ebert indices, are Lorenz Consistent.

As far as Principle of Addition is concerned only mean deviation, variance and the indices of Elteto and Frigyes do not satisfy this condition.

The lower limit of all the measures is zero, while there are only few indices that have meaningful upper limit. Elteto and Frigyes indices, Kakwani index, Gini coefficient, generalized Gini indices, Delton’s index and Atkinson’s indices are the few measures that have upper limit of one. It may, however, be noted that any measure with finite lower and upper limits can be converted to a [0, 1] range through an appropriate linear transformation.

Coming now to the decomposition of inequality measure, variance, coefficient of variation and Gini coefficient are the only three measures that are decomposable both in additive and non-additive forms, while Theil’s two measures, generalized entropy indices and Ebert’s indices are only additively decomposable.

Table 1: Comparison of Inequality Measures in Terms of Properties and Limits

Inequality Measure	Pigou-Dalton Condition		Principle of Population & Symmetry	Income Scale Independence	Lorenz Consistent	Principle of Addition	Decomposability		Defined Limits	
	Normal	Stronger					Additive	Non Additive	Lower	Upper
Range	No	No	Satisfy	Satisfy	No	Satisfy	No	No	Zero	n
Mean Deviation	No	No	Satisfy	No	No	No	No	No	Zero	$2\bar{Y}(n-1)/n$
Relative Mean Deviation	No	No	Satisfy	Satisfy	No	Satisfy	No	No	Zero	$2(n-1)/n$
Variance	Satisfy	No	Satisfy	No	No	No	Satisfy	Satisfy	Zero	$\bar{Y}^2(n-1)$
Coefficient of Variation	Satisfy	No	Satisfy	Satisfy	Satisfy	Satisfy	Satisfy	Satisfy	Zero	$\sqrt{n-1}$
Variance of Log-incomes	No	No	Satisfy	Satisfy	No	Satisfy	No	No	Zero	No limit
Elteto & Frigyes Indices	No	No	Satisfy	Satisfy	No	No	No	No	Zero	One
Kakwani Index	Satisfy	Satisfy	Satisfy	Satisfy	Satisfy	Satisfy	No	No	Zero	One
Schultz Index	No	No	Satisfy	Satisfy	No	Satisfy	No	No	Zero	$(n-1/n)$
Gini Coefficient	Satisfy	No	Satisfy	Satisfy	Satisfy	Satisfy	Satisfy	Satisfy	Zero	One
Generalized Gini Indices (Kakwani)	Satisfy	No	Satisfy	Satisfy	Satisfy	Satisfy	No	No	Zero	One
Generalized Gini Indices (Donaldson-Weymark)	Satisfy	No	Satisfy	Satisfy	Satisfy	Satisfy	No	No	Zero	One
Generalized Gini Indices (Yitzhaki)	Satisfy	No	Satisfy	Satisfy	Satisfy	Satisfy	No	No	Zero	One
Generalized Gini Indices (Chotikapanich-Griffith)	Satisfy	No	Satisfy	Satisfy	Satisfy	Satisfy	No	No	Zero	One

Theil's First Measure	Satisfy	Satisfy	Satisfy	Satisfy	Satisfy	Satisfy	Satisfy	No	Zero	$\ln(n)$
Theil's Second Measure	Satisfy	Satisfy	Satisfy	Satisfy	Satisfy	Satisfy	Satisfy	No	Zero	No limit
Generalized Entropy Indices	Satisfy	Satisfy	Satisfy	Satisfy	Satisfy	Satisfy	Satisfy	No	Zero	$\frac{1}{c(c-1)}\left(n^{c-1}-1\right)$
Dalton's Indices	Satisfy	Satisfy	Satisfy	No	No	Satisfy	No	No	Zero	One
Atkinson's Indices	Satisfy	Satisfy	Satisfy	Satisfy	Satisfy	Satisfy	No	No	Zero	One
Ebert's Indices	Satisfy	Satisfy	Satisfy	Satisfy	Satisfy	Satisfy	Satisfy	No	Zero	No limit

Different measures have different responses to the transfers of income from one income unit or group to another. Some measures are more sensitive to income transfers at upper income tail, while others are more sensitive to transfers at the lower end of the distribution. The measures that are more sensitive to transfers of income between rich classes are called 'alpha type' measures. The coefficient of variation falls in this category. The measures that are more sensitive to transfers of income between poor classes are called 'gema type' measures. Variance of logarithms and Theil entropy measure fall in this group. The measures, which are more sensitive to transfers of income in the middle-income range, are called 'beta type' measures. This category includes Gini coefficient. Finally, note that the sensitivity of all pure normative measures and generalized indices depends on the values of the relevant sensitivity parameters[14].

5. Summary

It appears that although there is a large literature available on the measures of income inequality, but there are only few measures that meet the criteria of desirable properties that an ideal inequality measure should fulfill. As far as the measurement of income inequality is concerned, coefficient of variation, Kakwani index, Gini coefficient, generalized Gini indices, Theil's two measures, generalized entropy indices, Atkinson's indices and Ebert's indices can be considered as the best measures. However, among these measures Kakwani index, generalized Gini indices and Atkinson's indices are not decomposable additively or non-additively, while Theil's two measures and generalized entropy indices

[14] For the empirical estimates of these inequality measures see Idrees M and E. Ahmad (2010), Idrees M. and Ahmad E. (2012) and Idrees M. (2012).

and Ebert's indices are decomposable additively only. Thus for a thorough analysis of income inequality coefficient of variation and Gini coefficient are the only available measures that posses all the desirable properties. Finally, since sensitivity of an inequality measure to the location of income transfers also varies across various measures, hardly any measure can serve all purposes and it is desirable to employ more than one measure in an empirical analysis of income inequality.

In nutshell it is concluded that each inequality measure looks at income inequality from different dimension. The selection of an appropriate inequality measure depends upon the objective of researcher. If objective is merely to measure income inequality then Gini coefficient is the most appropriate measure, if the objective is decomposition then along with Gini Coefficient, generalized entropy measures are the best choices and if the objective is to incorporate value judgment then Atkinson's indices are the best choice.

References

Anand, S. (1983). *Inequality and Poverty in Malaysia, Measurement and Decomposition.* Oxford University Press, International Bank for Reconstruction and Development, Washington, D.C. U.S.A.

Atkinson, A. B. (1970). On the measurement of inequality. *Journal of Economic Theory, 2*, 244-263.

Bhatacharya & Mahalanonis (1967). Regional disparities in household consumption in India. *Journal of the American Statistical Association, 62*, 143-161.

Chakravarty, S. R. (1988). Extended Gini indices of inequality. *International Economic Review, 29*, 147-56.

Chotikapanich, D., & Griffiths, W. (2001). On the calculation of the extended Gini Coefficient. *Review of Income and Wealth, 47*, 541-547.

Cowell, F. A. (1980). On the structure of additive inequality measures. *Review of Economic Studies, 47*, 521-531.

Cowell, F. A. (2000). *Measuring Inequality,* Third Edition, Oxford University Press.

Dagum, C. (1997). A new approach to the decomposition of Gini income inequality ratio. *Empirical Economics, 22*, 515-531.

Dalton, H. (1920). The measurement of the inequality of incomes. *Economic Journal, 30*, 348-61.

Das, T. & Parikh, (1982). Decomposition of inequality measures and a comparative analysis. *Empirical Economics, 7*, 23-48.

Donaldson, D., & Weymark, J. A. (1980). A single-parametric generalization of the Gini indices of inequality. *Journal of Economic Theory, 22*, 67-86.

Donaldson, D., & Weymark, J. A. (1983). Ethically flexible Gini indices for income distributions in the continuum. *Journal of Economic Theory, 29*, 353-358.

David, H. A. (1968). Gini's mean difference rediscovered. *Biometrika, 55*, 573-575.

Ebert, U. (1999). Dual decomposable inequality measures. *Canadian Journal of Economics, 32*, 234-246.

Elteto, U., & Frigyes, E. (1968). New income inequality measures as efficient tools for causal analysis and planning. *Econometrica, 36*, 383-396.

Field, G., & Fie, (1978). On inequality comparison. *Econometrica, 46*, 313-316.

Foster, J. E., & Shneyerov, A. A. (2000). Path independent inequality measures. *Journal of Economic Theory, 91*, 199-222.

Gini, C. (1912). Measurement of inequality of incomes. *Economic Journal, 31*, 124-126.

Gini, C. (1921). *Variabilitae Mutabilita Bolonga:* Tipografia di Paola Cuppini.

Idrees, M. (2010). An analysis of district level earning inequalities in Pakistan. *Journal of Business & Economics, 4(1)*, 124-142.

Idrees, M & Ahmad, E. (2010). Measurement and decomposition of consumption inequality in Pakistan. *The Lahore Journal of Economics, 15(2)*, 97-1112.

Idrees, M & Ahmad, E. (2012). An analysis of international income inequality. *Forman Journal of Economic Studies, 8*, 1-12.

Kakwani, N. C. (1980a). *Income inequality and poverty: Methods of Estimation and Policy Applications,* Oxford University Press @ International Bank for Reconstruction and Development, Washington, D.C. U.S.A.

Kakwani, N. C. (1980b). On a class of poverty measures. *Econometrica, 48*, 437-446.

Kendall, M. G., & Stuart, A. (1963). *The Advanced Theory of Statistics* 1, *Distribution Theory,* Second Edition, London: Griffin.

Kondor, Y. (1971). An old-new measures of income inequality. *Econometrica, 6*, 1041-1042.

Kuan, Xu, (2003). How has the literature on Gini's index evolved in the past 80 years?," *Working Papers Archive* Department of Economic, Dalhousie University.

Litchfield, J. A. (1999). *Inequality: Methods and Tools*. World Bank's Web site on Inequality, Poverty and Socio-economic Performance, www.worldbank.org/poverty/inequality/index.html

Lorenz, M. C. (1905). Methods of measuring the concentration wealth. *Journal of The American Statistical Association,* 9, 209-219.

Money, L. G. (1905). *Riches and Poverty,* London: Methew & Company Ltd.

Mookherjee, D., & Shorrocks, A. F. (1982). A decomposition analysis of the trend in UK income inequality. *The Economic Journal, 92*, 886-902.

Pyatt, G. (1976). On the interpretation and disaggregation of Gini coefficients. *Economic Journal, 86*, 243-255.

Rao, V. M. (1969). Two decompositions of concentration ratio. *Journal of the Royal Statistical Society, 132.*

Schultz, R. R. (1951). On the measurement of income inequality. *American Economic Review, 41*, 107-122.

Sen, A. (1973). *On Economic Inequality,* Oxford: Clarendon Press, New York.

Shorrocks, A. F. (1980). The class of additively decomposable inequality measures. *Econometrica, 48*, 613-625.

Shorrocks, A. F. (1982). Inequality decomposition by factor components. *Econometrica, 50*, 193-211.

Shorrocks, A. F. (1983). Ranking Income Distributions. *Econometrica, 50*, 3-17.

Shorrocks, A. F. (1984). Inequality decomposition by population sub-groups. *Econometrica, 52,* 1369-1385.

Silber, J. (1989). Factor components, population subgroups and the Gini index of inequality. *Review of Economics and Statistics, 71*, 107-115.

Theil, H. (1967). *Economics and Information Theory* (North-Holland, Amsterdam).

Turroni, B. C. (1910). Di un indice misuratore della disuguaglianza della ricchezza. *In Studi in Onore di onere B*. Brugi, Palermo.

Vega. & Urrutia, A. M. (2003). A new factorial decomposition for the Atkinson measure. *Economic Bulletin, 4*, 1-12.

Yitzhaki, S. (1983). On an extension of the Gini inequality index. *International Economic Review, 24*, 617-628.

Yitzhaki, S. (1991). Calculating jackknife variance estimators for parameters of the Gini method. *Journal of Business & Economic Statistics*, *9,* 235-239.

Yitzhaki, S. (1994). Economic distance and overlapping of distributions. *Journal of Econometrics, 61*, 147-159.

Appendix: A

Table A1: Additive Decomposition of Inequality Measures

Measure	Formula	Within Component	Between Component
Theil First measure	$T_1 = \frac{1}{n}\sum_{i=1}^{n}\left(Yi/\bar{Y}\right)\ln\left(Yi/\bar{Y}\right)$	$\sum_{k=1}^{K} s_k . T_1^k$	$\sum_{k=1}^{K} s_k \ln\left(\bar{Y}_k/\bar{Y}\right)$
Theil Second measure	$T_2 = \frac{1}{n}\sum_{i=1}^{n}\left[\ln\left(\bar{Y}/Y_i\right)\right]$	$\sum_{k=1}^{K} p_k . T_2^k$	$\sum_{k=1}^{K} p_k \ln\left(\bar{Y}/\bar{Y}_K\right)$
Generalized Entropy measure	$I_c = \frac{1}{n}\frac{1}{c(c-1)}\sum_{i=1}^{n}\left[\left(Yi/\bar{Y}\right)^c - 1\right]$	$\sum_{k=1}^{K}\left[\left(s_k\right)^c\left(p_k\right)^{1-c}\right] I_c^k$	$\dfrac{\sum_{k=1}^{K}\left[p_k\left(\frac{\bar{Y}_k}{\bar{Y}}\right)^c - 1\right]}{c(c-1)}$
Ebert's Measure	$E_{(\varepsilon)} = \dfrac{\bar{Y}}{\left[\frac{1}{n}\sum_{i=1}^{n}(Yi)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}} - 1,$ $\varepsilon > 0$ $\varepsilon \neq 1$ $E_{(\varepsilon)} = \dfrac{\bar{Y}}{\prod_{i=1}^{n}(Yi)^{1/n}} - 1,\ \varepsilon = 1$	$\sum_{k=1}^{K} w_k\left[E_{(\varepsilon)}^k\right]$ $\sum_{k} w_k\left[E_{(\varepsilon)}^k\right]$	$\sum_{k=1}^{K} p_k\left(\dfrac{\sum_k Y_k^{1-\varepsilon}}{\sum Y_k^{1-\varepsilon}}\dfrac{n}{n_k}\right)^{\frac{1}{1-\varepsilon}}$ $\sum_{k} p_k\left(\dfrac{\prod_k\left(Y_k\right)^{\frac{1}{n_k}}}{\prod Y^{\frac{1}{n}}} - 1\right)$
Gini Coefficient	$G = \frac{1}{2n^2\bar{Y}}\sum_{i-1}^{n}\sum_{j-1}^{n}\left\lvert Y_i - Y_j\right\rvert$	$\sum_{j=1}^{k} p_j s_j G_{jj}$, where $G_{jj} = \dfrac{\sum_{i=1}^{n_j}\sum_{r=1}^{n_j}\left\lvert Y_{ji} - Y_{jr}\right\rvert}{2n_j^2\bar{Y}_j}$	$\sum_{j=1}^{k}\sum_{h=1}^{k} p_j s_h G_{jh}$, $i \neq h$, where $G_{jh} = \dfrac{\sum_{i=1}^{n_j}\sum_{r=1}^{n_h}\left\lvert Y_{ji} - Y_{hr}\right\rvert}{n_j . n_h\left(\bar{Y}_j + \bar{Y}_h\right)}$ Net between: $\sum_{j=2}^{k}\sum_{h=1}^{j-1} G_{jh}\sigma_{jh} D_{jh}$ Trans-variation:

			$\sum_{j=2}^{k}\sum_{h=1}^{j-1} G_{jh}\sigma_{jh}\left(1-D_{jh}\right)$

$p_k = n_k / n$, $s_k = n_k \overline{Y}_k / n\overline{Y}$, $w_k = \left(Y_{e(k)}/Y\right)\left(n_k/n\right)$, $\sigma_{jh} = p_j s_h + p_h s_j$,

$D_{jh} = \left(d_{jh} - p_{jh}\right)/\left(d_{jh} + p_{jh}\right)$, $d_{jh} = \sum_{i=1}^{n_j} p_{ji} \sum_{r|Y_{ji}>Y_{kr}} \left(Y_{ji} - Y_{hr}\right) p_{hr}$,

$p_{jh} = \sum_{i=1}^{n_j} p_{ji} \sum_{r|Y_{ji}<Y_{hr}} \left(Y_{ji} - Y_{hr}\right) p_{hr}$