Money Demand Function for Pakistan

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Abstract

The main objective of this study is to empirically estimate the long run money demand function for Pakistan using time series data. For this purpose we used annual data from 1953 to 2003. The results of unit root analysis have suggested that both log of nominal GDP and log price are possibly I (2) variables. The results of unit root with structural break for real M_1 and M_2 have suggested that log of real M_1 is fractionally integrated where as log of real M_2 is trend stationary. Results of I(2) co-integration analysis have suggested that there are some I(2) trends in the model with nominal variables hence in order to avoid complications involved in the analysis of I(2) trends we transformed our model in real variables. We found one co-integration relation for both M_1 and M_2. The sign of the estimated coefficients for GDP and interest rate in M_1 money demand function are according to theory but coefficient of interest rate has wrong sign for M_2 money demand relation but it is statistically insignificant. We accepted the hypothesis that both real GDP and interest rate are weakly exogenous in long run money demand relation for both M_1 and M_2. Demand for money for both M_1 and M_2 are found to be inelastic with respect to interest rate which is quite obvious for an underdeveloped country like Pakistan where financial markets are underdeveloped, infrastructure is poor and information system is still very slow.

I. Introduction

The main objective of this study is to estimate long run stable money demand function for Pakistan. Some research has already been done for Pakistan on this issue but different studies have arrived at different conclusions. This is due to the fact that different studies have used different data sets and different

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methodologies. In this study we would like to estimate money demand function for longer time series and by using recent advances in time series analysis. We have used 51 annual observations from 1953 to 2003 for estimation.

First we did univariate analysis of the single series to know the order of integration for both nominal and real variables. We have applied ADF and KPSS unit root test by allowing structural break in the trend function for real series as there is quite visible structural break due to oil price shock of the history (see Perron (1989), Busetti et al (2001).

We have estimated long run demand function for narrow money (M_1) and broad money (M_2) in dynamic vector auto regressive (VAR) models by using Johansen cointegration framework (Johansen (1988)). Other variables in the model are GDP, price and interest rate. We will do I (2) cointegration analysis in order to make sure that there are no I (2) trends in our model.

The results of ADF unit root test have shown that log of price and log of nominal GDP are I (2) variables where as KPSS test has not supported this evidence. The results of unit root analysis for real variables have suggested that M_1 is difference stationary when we have not allowed structural break where as it is fractionally integrated when we allowed structural break in the tests. M_2 is found to be trend stationary even when we have not allowed for structural break but the results with structural break strongly confirmed that this series is trend stationary.

Results of I (2) analysis for modelling both nominal M_1 and M_2 have shown that there are some I (2) trends in the model. In order to avoid the difficulties involved in I (2) analysis we then transformed our nominal variables into real. The result of I (2) analysis then confirmed that there are no I (2) trends in model with real variables.

We found one cointegration relation in modelling both M_1 and M_2. The sign of the coefficients in M_1 money demand relation are according to theory where as for M_2 money demand relation interest rate has the wrong sign but it is found to be statistically insignificant. The test of hypothesis for adjustment matrix have shown that both GDP and interest rate are weakly exogenous in the long run money demand relation. We also accepted the hypothesis that both M_1 and M_2 inelastic with respect to interest rate. This result is quit obvious for an underdeveloped country like Pakistan where most of the people are living on subsistence level and financial market is imperfect. Rest of the study is organized as follows. Section II, provides review of relevant studies on this issue pertaining to Pakistan. We discussed theoretical background on money demand in section III. Besides, data and different variables included in our model are also discussed in this section. Section IV consists of Unit root analysis. Cointegration tests are discussed in section V. Moreover, multivariate dynamic analysis for estimating long run money demand is also analyzed in this section. The conclusion of the study is given in section VI.

II. Literature Review

A lot of empirical research has been done for estimating money demand function for different countries. In this section we would like to mention some of the earlier studies and their main findings about the estimation of money demand function for Pakistan. The first important study for Pakistan was done by Manga (1979). This study only used 14 annual observations and results he obtained were not according to theory and were not reliable due to such a small sample.

Khan (1980) did his study on larger sample starting from 1960 to 1978. The study tried to see the effect of inflation and monetization on money demand. It was found that inflation has no effect on money demand before 1971 but it has significant effect after that period due to higher inflation. Khan (1981) estimated money demand function for different definitions of money by using the same data but he arrived at the same results.

Cornelisse (1989) estimated the money demand function for Pakistan by using monthly data from 1975 to 1989. The data on GDP on monthly basis was not available hence he divided the annual GDP to 12 parts to account for this problem. The study also used broad and narrow definition of money to estimate disaggregated money demand function. The author found better results for broader definition of money as compared to narrow money.

Using the data from 1951 to 1991 Hossain (1994) performed cointegration analysis to find stable money demand function. The results showed that stable money demand function do not exist for broad definition of money. Khan (1994) found a stable relationship between broad money, real income and medium term interest rate by using quarterly data from 1971.3 to 1993.2. The study also found that money demand is not cointegrated with short run interest rate and inflation rate. M1 money was found to be cointegrated with real income, real interest rate and inflation rate.

The review of above literature suggests that there exist huge differences in the results for money demand function in Pakistan. The reason could be that these different studies have used different data set and different methodology. In this study we would rather like to use advance econometric techniques to search for stable money demand function.

III. Theoretical Background

There exists huge and diverse literature on theories of money demand. These different theories directly or indirectly suggest that real money demand depends on real transactions volume and nominal interest rate. Nominal interest rate represents opportunity cost of holding money. The basic money demand function can be summarised in the following functional from[2]:

$$\frac{M}{P} = f\left(\frac{Y}{P}, i\right)$$

Where M is the aggregate demand of money, "Y" is Gross domestic product, "P" is the price index which is used to find the real money demand and Real GDP and "i" is the interest rate. The long run stable relationship in linear form can be written as follows:

$$m_t = \beta_1 + \beta_2 y_t + \beta_3 i_t$$

where: m_t = Log of Real money balances
y_t = Log of Real GDP
i = Interest Rate

Theoretically we expect $\beta_2 > 0$ and $\beta_3 < 0$ for meaningful money demand relation (for more detail see for example Ericsson and Sharma (1996)). $\beta_2 = 1$ is consistent with quantity theory of money and $\beta_3 = 0$ will exclude the role of interest rate in the determination of money demand.

Some studies have also used expected inflation as an explanatory variable for explaining money demand. This is typically done for the countries where financial market is not well developed or there is very high inflation rate (see Choudhry(1995a) and Choudhry (1995b)). Arestis et al (1991) have argued that in developing countries which do not have alternative financial assets to money, nominal interest rates can be viewed as own-rate of money and expected inflation rate is the return on real assets.

3.1. Data and Variables

We used M_1 and M_2 definition of the money. M_1 is the narrow definition of money which is defined as the money stock which is readily available in everyday transactions and it consists of sum of currency outside deposit money banks and demand deposits other than those of the central governments. M_2 is the broad definition of money stock with less liquid assets and components of this

[2]. *For detail survey of literature on money demand see for example Sriram (1999).*

stock of money are M_1 plus time, saving and foreign currency deposits of resident sectors other than central government[3]. Gross domestic product (GDP) represents the transactions volume. In the literature long run interest rate has been used to capture the effect of opportunity cost of holding money but in under develop country like Pakistan where inflation is very high, financial markets are imperfect and real interest rate is sometimes close to zero or even negative hence we would rather like to use short run interest rate which is also known as inter bank call money rate. For prices we used GDP deflator because data of CPI was not available for some starting years.

The data on different definitions of money is available on monthly and quarterly basis but it is not available for GDP series. Instead of GDP some studies have used index of industrial production as an approximation. For our study we would rather like to use annual data which is available from 1953 to 2003 for Pakistan. By using annual data we can avoid a lot of seasonal variations but we will have fewer observations as compared to quarterly data. M_1, M_2 and and GDP are deflated by GDP deflator to find the real variables. The data are been downloaded from the website of International Financial Statistics[4]. The graphs of the nominal and real series are shown in figure A1 and A2 in the Appendix respectively.

IV. Unit Root Analysis

We would like to use the Pantula principle (Pantula (1989)) in order to know the integration order to the single series. According to this principle first we will difference the series until the unit root is rejected by using different test. Then we will apply unit root tests on the series with one less difference than in the previous test. This procedure will continue until unit root is rejected.

A series is stationary if the roots of the characteristic equation lie inside the unit root circle or roots of lag polynomial lie out side the unit circle. There are different test proposed by the theory and each test has its own pros and cons. These tests include Dicky Fuller test (Fuller (1976), Dickey & Fuller (1979)), Augmented Dicky-Fuller (Dicky & Fuller (1981)), Phillips Perron test (Phillips (1987), Perron (1988), Perron (1989)), KPSS test (Kwiatkowski, et al (1992)).

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[3]. These definitions have been used in the literature and we have taken from International Financial Statistics.

[4]. Link to International Financial Statistics can be found on the homepage of IMF official website www.imf.org

4.1. Dickey Fuller and Augmented Dicky Fuller Test

Dicky Fuller test is proposed to test for unit root in first order auto regressive model i.e AR (1) with the assumption that errors are white noise. The basic regression of this test can be written as follows:

$$Y_t = m + \alpha Y_{t-1} + \varepsilon_t \qquad (1)$$

In the above regression if α < 1, where α is actually the characteristic root of the above difference equation or reciprocal of the root of lag polynomial, then Y series will be stationary. The null hypothesis is that root is one and against the alternative that it is less than one, so we are testing non-stationarity in the null hypothesis against the alternative of stationarity. Hence under the null hypothesis the series is random walk and its distribution of disturbances is non standard. The usual standard procedure can not be applied instead Dicky-Fuller distribution can be used and this distribution depends upon the deterministic parts of the model. Above analysis requires that the disturbances are white noise and if it is not the case then we will have to use other tests. One possibility could be to use Augmented Dicky-Fuller test.

Augmented Dicky-Fuller test (Dicky & Fuller (1981)) is just the extension of the simple Dicky-Fuller test to make the disturbances white noise by including more lags of the dependent variable. There is another advantage of using ADF test over simple DF test that it can be used to test unit root in higher order auto regressive scheme. General ADF regression can be written as follows:

$$\Delta Y_t = m + \gamma Y_{t-1} + \beta_1 \Delta Y_{t-1} + \beta_2 \Delta Y_{t-2} + \ldots\ldots\ldots + \beta_p \Delta Y_{t-p} + \varepsilon_t \qquad (2)$$

In this formation testing for unit root means that we are testing for γ = 0 against the alternative that it is less then zero. The decision about how many lag differences to be included in the regression can be based on the model selection criterion, for example AIC criterion or a sequential testing procedure in which insignificant lags can be eliminated.

4.2. ADF Test for Series with Structural Breaks

Perron (1989) has proposed a unit root test for the series with structural break. He showed that standard unit root test failed to reject the null hypothesis of difference stationarity against the alternative of trend stationarity if there are structural breaks in the series with known time of the breaks. He suggested three different models to incorporate the changes in the trend function. Model (1) allows for the shift in intercept of the trend function and he referred it as the “crash model”. Model (2) allows for the change in the slope of the trend function and he referred this model as the “changing growth”. Model (3) allows for the change in both intercept and slope of the trend function which is referred as sudden change and followed by a different growth path. The functional forms of the three models are as follows:

Model (1)

$$y_t = \alpha + \delta T + \psi D + u_t \text{ and } u_t = \beta_1 u_{t-1} + \beta_2 u_{t-2} + ... + \beta_p u_{t-p} + \varepsilon_t$$

$$\Delta Y_t = m_1 + \psi_1 D + \delta_1 T + \gamma_1 Y_{t-1} - \beta_1 \Delta Y_{t-1} - \beta_2 \Delta Y_{t-2} - ....$$
$$-\beta_p \Delta Y_{t-p+1} + \varepsilon_t \text{ ....(3)}$$

Model (2)

$$y_t = \alpha + \delta T + \phi D_t + u_t \text{ and } u_t = \beta_1 u_{t-1} + \beta_2 u_{t-2} + ... + \beta_p u_{t-p} + \varepsilon_t$$

$$\Delta Y_t = m_2 + \phi_2 D_t + \delta_2 T + \gamma_2 Y_{t-1} - \beta_1 \Delta Y_{t-1} - \beta_2 \Delta Y_{t-2} - ....$$
$$-\beta_p \Delta Y_{t-p+1} + \varepsilon_t \text{ ....(4)}$$

Model (3)

$$y_t = \alpha + \delta T + \psi D + \phi D_t + u_t \text{ and } u_t = \beta_1 u_{t-1} + \beta_2 u_{t-2} + ... + \beta_p u_{t-p} + \varepsilon_t$$

$$\Delta Y_t = m_3 + \delta_3 D + \theta_3 T + \phi_3 D_t + \gamma_3 Y_{t-1} - \beta_1 \Delta Y_{t-1} - \beta_2 \Delta Y_{t-2} - ....$$
$$-\beta_p \Delta Y_{t-p+1} + \varepsilon_t \text{ ....(5)}$$

Where D = 0 for $T < T_B$ and D = 1 for $T \geq T_B$
$D_t = 0$ for $T < T_B$ and $D_t = T - T_B$ for $T \geq T_B$

Where T_B is the time when break occurred.

The test of unit root in all the above models corresponds to test the hypothesis that coefficient of Y_{t-1} is zero. The distribution of test statistic depends on deterministic of the models as in the case of tests without the break but now in this case it also depends upon the time of the break. Perron (1989) defined a parameter "λ" know as break fraction to capture the timing of the break and it is defined as the ratio between pre break sample size with total sample size.

4.3. KPSS test

All the tests described above test for unit root against the alternative of level or trend stationarity. Kwiatkowski et al (1992) proposed an LM test in which we test for stationarity against the alternative of unit root. This test uses the following model to test for unit root:

$y_t = \alpha + \delta t + r_t + \varepsilon_t$.............(6)

Where $r_t = r_{t-1} + u_t$ $\quad u_t$ is iid $(0, \sigma_u^2)$

Ho: $\sigma_u^2 = 0$ (y_t is stationary after detrending, demeaning or both)

H1: $\sigma_u^2 \neq 0$ (y_t is non stationary)

KPSS test statistics $= T^{-2} \sum_{t=1}^{t} S_t^2 \Big/ s^2(l)$

Where $s^2(l)$ is the long run variance

and $S_t = \sum_{i=1}^{t} \varepsilon_i \qquad t = 1,2,.......,T$

Normally this test is used together with other tests to get conclusive result about the stationarity of the series. Baillie et al (1996) summarized four possible outcomes if KPSS test is used together with ADF and PP test which are described as follows:

- Rejection of the hypothesis by ADF and PP test and failure to reject the hypothesis by KPSS test is viewed as strong evidence of covariance stationary process.
- Failure to reject hypothesis by ADF and PP test and rejection by KPSS test is viewed as strong evidence of unit root process.
- Failure to reject the hypothesis by ADF, PP and KPSS test is viewed as insufficiency of information from the data.
- Rejection of hypothesis by ADF, PP and KPSS test indicates the possibility of fractional integration.

4.4. KPSS test with structural breaks

Busetti et al (2001) modified KPSS test for presence of random walk component in a stationary or trend stationary time series by allowing for structural break in the series. They also simulated the distribution of the modified test statistic. Test statistic is exactly the same as in the above KPSS test but now we have to use residuals from model that allows for structural break by using appropriate dummy variables for shift in level or shift in the slope of the trend function. The general model can be written as follows:

$$y_t = \alpha + \delta t + r_t + \alpha_d D + \delta_{dt} D_t + \varepsilon_t \text{...........}(7)$$

where $D = D_t = 0$ before break and $D = 1$ and $D_t = T - T_B$ after the break

The choice of dummy variable D and D_t depends upon the nature of the break as we already discussed in section 4.2.

4.5. Results of Unit Root Tests

The results of unit root tests described above are reported and discussed in this section.

Table: 1. Result of Unit Root for Nominal Variables

Variable	Level		First Difference		Second Difference		Status
	ADF test statistic	**KPSS test Statistic**	**ADF test statistic**	**KPSS test Statistic**	**ADF test Statistic**	**KPSS test statistic**	
Nominal Money M_1	-3.08 (3) C, T	0.368	-5.91 (0) C	0.490	-8.27 (1) C	0.034	I (1)
Nominal Money M_2	-2.59 (1) C, T	0.336	-5.15 (0) C	0.332	-6.59 (2) C	0.024	I(1)
Nominal GDP	-2.27 (1) C, T	0.335	-2.51 (3) C	0.378	-8.68 (0) C	0.069	I(2)
Price	-2.56 (1) C, T	0.270	-2.79 (3) C	0.288	-6.54 (1) C	0.055	I(2)
Interest Rate	-1.87 (0) C	1.161	-7.18 (0)	0.371	-9.52 (1)	0.113	I(1)
Critical Value 5%	3.50 (C, T)	0.146 (C, T)	2.92 (C)	0.463 (C)	2.92 (C)	0.463 (C)	

- *Number in the parenthesis represents number of lags included in the ADF regression to make the error white noise*
- *In KPSS test automatic lag selection criterion is used for selecting optimal lags for calculating long run variance*
- *C and T represent constant and trend respectively as deterministic of the model.*

4.5.1 Unit Root Analysis of Nominal Variables

The graphs of the nominal series are shown in the appendix A.1. The results of the unit root analysis for nominal variables are reported in table (1). Following the Pantula principle first we applied unit root tests on the second differences for all the series and results have shown that second differences of all the series are stationary. In second step we applied unit root tests on first differences. For M_1 series both ADF and KPSS test rejected their respective hypothesis hence there is possibility of fractional integration in first difference of the series. First difference of M_2 is turned out to be stationary by both ADF and KPSS test. The unit root test results on the first differences of Log of nominal GDP and log price have shown that there is insufficiency of information from the data since both ADF and KPSS test have accepted their respective hypothesis. First difference of interest rate is found to be stationary. Then we proceed to test for unit root on level of the different series. We included trend in the regression for all the series except for interest rate. All the series except interest rate are found to difference stationary in level where as interest rate is found to be non-stationary. The above discussion suggests that except nominal GDP and price all the series are integrated of order one where as these two series could possible be I (2).

4.5.1 Unit Root Analysis of Real Variables

In order to avoid the complications involved in analysis of I (2) variables we then transformed nominal variables into real variables by dividing nominal variables with price. The analysis of real variable is more interesting and important so we will discuss the unit root analysis of individual series separately and in more detail. Following the Pantula principle first we applied unit root on first differences and first differences of all the series are found to be stationary hence we proceeded by applying unit root test on level of each series.

4.5.2 Log of Real Money M_1 and M_2

If we see the graph of real money M_1 and M_2 in figure 2, there is clearly a structural break at the year 1974. This break is well known due to the oil price shock[5] of the history that affected almost all the countries. It can be seen that the level of M_1 series is shifted downward where as slope of it is increased by this structural change. For M_2 only level is shifted downwards but the slope of the series is constant. Hence in M_1 series we allowed two dummies, one for the change in the intercept and one for the change in the slope of the trend function where as in M_2 series only intercept dummy is allowed to capture level shift. Model (3) is estimated for unit root test on M_1 and Model (1) is estimated for M_2. The critical values of the test statistic depend on the value λ which is equal to 0.4 in our study. The series are difference stationary under the null and trend

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[5] *The structural break could also be due to the separation of the Pakistan into East Pakistan (Bangladesh) and West Pakistan (Pakistan) in 1971.*

stationary against the alternative hypothesis. If D and D_t is not included then this is usual Augmented Dicky Fuller regression. The results of unit root (with and without structural break) are reported in table 2. For M_1 series ADF test failed to reject unit root test and KPSS test has rejected the hypothesis of stationarity hence there is strong evidence of difference stationarity. For M_2 series the null hypothesis of the ADF test is rejected at 5% level (but accepted at 1% level) where KPSS test fails to reject the null hypothesis even at 1% level of significance hence there is strong evidence of trend stationarity in M_2 at 5% level (but not at 1% level of significance).

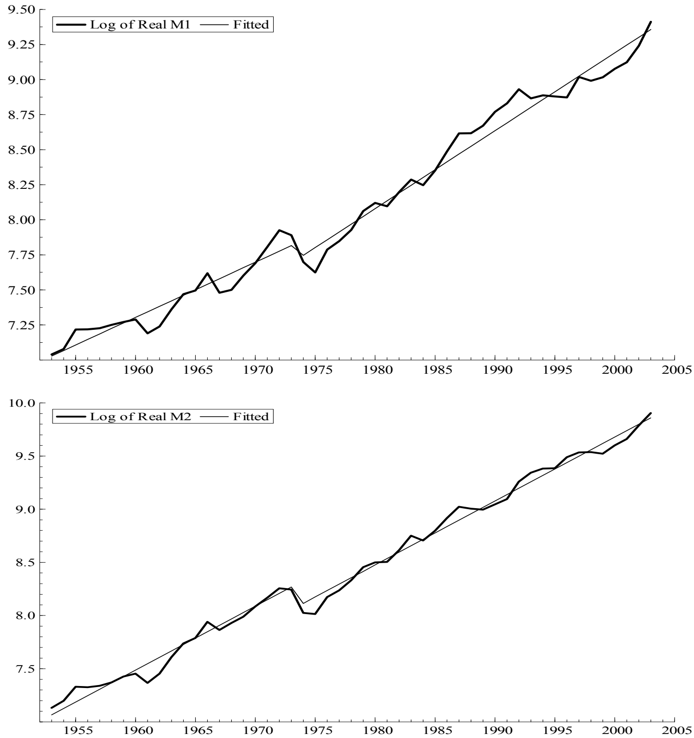

Figure 2. *Graph of the actual series with deterministic fitted trend*

When we allowed structural break in M_1 series then this series turned out to be fractionally integrated since both ADF and KPSS test have rejected their respective hypothesis. Results with structural break in M_2 confirmed that it is trend stationary and now both ADF and KPSS tests have rejected and failed to reject the null hypothesis respectively at 1% level of significance. This result is opposite to the earlier findings where both M_1 and M_2 definition of money were found to be difference stationary (Hossain (1994)). This difference could be due to different reasons but results of this study are more reliable since we have used longer time series and applied better technique since ADF without structural break has less power because it takes known structural break as a noise into the process Perron (1989).

Table: 2. Result of Unit Root Test for Real Money Supply

	Series	**ADF**	**KPSS**				**Status**
			L= 0	**L= 4**	**L= 8**	**Auto[6] L = 4**	
Without Structural Break	M_1	-0.296 (-2.91)	0.492	0.184	0.146	0.184	Fail to reject by ADF test and rejected by KPSS hence there is strong evidence of difference stationary.
	M_2	-0.434 (-3.76)	0.244	0.104	0.106	0.104	Rejection by ADF and Failure to reject by KPSS. Hence Strong Evidence of Trend Stationarity
	Critical Value at 5%	-3.50				0.146	
With Structural Break	M_1	-0.499 (-4.25)	0.250	0.107	0.098	0.107	Rejection by ADF and rejection by KPSS is the evidence of Fractional Integration
	Critical Value at 5%	-4.22				0.066	
	M_2	-0.674 (-5.64)	0.203	0.095	0.106	0.095	Same result as above but now ADF is rejected even at 1 % level. Trend Stationarity
	Critical Value at 5%	-3.74				0.123	

() t-statistic is reported in parenthesis for ADF test

4.5.3 Unit Root Test on Log Prices

The graph of the series in figure (3) shows that after 1974 both intercept and slope of the trend function has been changed due to structural break. Results

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[6]. *Auto means automatic bandwidth selection procedure proposed by Newey and West (1994) as described by Hobijn et al. (1998, p.7) is used to determine maximum lags. In that case, a single value of the test statistic is produced, at the optimal bandwidth. Stata 7 used that procedure by default to select lag automatically.*

are reported in table 3. Hence we allowed two dummies one for change in the intercept and one for change in the slope of the trend function. Hence model (3) is estimated to test for unit root. ADF test has failed to reject the null hypothesis where KPSS test rejected the hypothesis at 5% hence there is strong evidence that series is difference stationary.

We then allowed for structural break in intercept and slope but still ADF test failed to reject the unit root hypothesis but now KPSS test also failed to reject the hypothesis hence this result is viewed as insufficiency of information from the data.

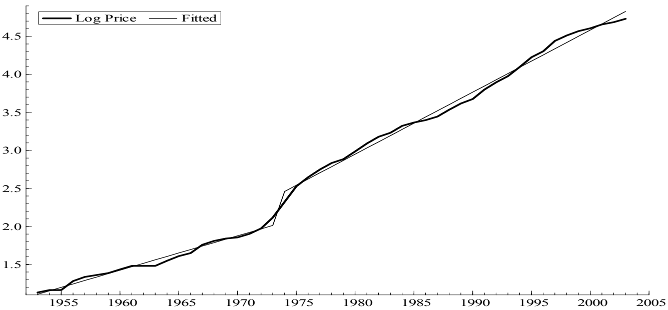

Figure 3. Graph of the actual values along with fitted trend

Table: 3. Result of Unit Root Test for Log Price (with and without Structural Break)

	ADF	**KPSS**				**Status**
Without Structural Break	0.116 (-2.56)	L= 0	L= 4	L= 8	Auto L= 4	Failure to reject by ADF and Rejection by KPSS. Hence Strong Evidence of Difference Stationarity
		0.663	0.164	0.121	0.164	
Critical Value at 5%	-3.50				0.146	
With Structural Break	-0.316 (-4.03)	0.203	0.093	0.124	0.088	Fail to reject by ADF and Rejection by KPSS is viewed as Difference Stationarity
Critical Value at 5%	-4.22				0.066	

4.5.4 Unit Root Test on Log of Real GDP

The graph of GDP in figure (4) also shows that intercept of the trend function is shifted little bit down after 1974; hence we applied the unit root test with structural break that allows for shift in the intercept of the trend function where as slope is almost constant after the break. The crash model (1) is estimated for this series as it was used in the case of unit root test on log of real money supply. The results are reported in table 4. ADF test is failed to reject the null hypothesis and KPSS has rejected the null hypothesis hence there is strong evidence of difference stationary. The results with structural break have given the same results.

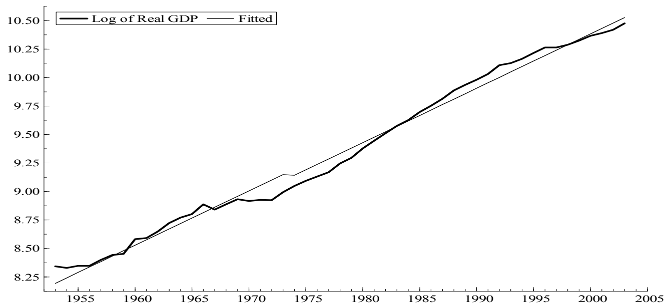

Figure 4. Graph of the actual values along with fitted trend

Table: 4. Result of Unit Root Test for Log of Real GDP (with and without Structural Break)

	ADF	KPSS				Status
Without Structural Break	-0.123 (-2.14)	L = 0	L = 4	L = 8	Auto L = 4	Failure to reject by ADF and Rejection by KPSS. Hence Strong Evidence of Difference Stationarity
		0.678	0.165	0.115	0.165	
Critical Value at 5%	-3.50				0.146	
With Structural Break	-0.0.088 (-1.49)	0.614	0.184	0.124	0.184	Same result as above Difference Stationarity
Critical Value at 5%	-3.74				0.123	

Unit root analysis above has suggested that real GDP and log price are difference stationary and interest rate is non stationary around level where as M_1 is difference stationary but with structural break it is fractionally integrated and M_2 is trend stationary.

V. Cointegration Analysis

The basic objective of the cointegration analysis is to look for stable long run relationship among variables. There are two methods to look for stable relation, first, Engle and Granger single equation static analysis proposed by Engle and Granger (1987), second, multiple equation dynamic analysis suggested by Johansen (1988).

5.1 Cointegration Analysis in Engle-Granger's Framework.

In this procedure cointegration exists if the linear combination of different I (1) variables is I (0). The methodology of this procedure is to regress variables in level and then apply unit root test on residuals obtained from the regression. If the residuals turn out to be stationary then the variables are said to be cointegrated and this linear combination could be interpreted as stable long run relationship. We have not used this procedure because it is based on very restrictive assumptions that all explanatory variables are exogenous.

5.2 Cointegration Analysis in Johansen's Framework.

Johansen cointegration analysis is based on dynamic VAR model (see Johansen (1988)). The unrestricted VAR model with lag order of "k" can be written as follows:

$$y_t = m_0 + \sum_{i=1}^{k} \pi_i y_{t-j} + \varepsilon_t$$

Where y_t is the vector of variables to be included in the model with dimension "p x 1" and y_{t-j} is the matrix of coefficients for vectors of lag variables. π_i is the matrix of coefficients for lag variables. ε_t is "p x 1" vector of stochastic random term distributed independently and identically. The vector error correction formulation of the model can be written as follow to represent short run and long run components of the model.

$$\Delta y_t = m_0 + \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + ... + \Gamma_{k-1} y_{t-k+1} + \varepsilon_t$$

Where $\Pi = 1 - \pi_1 - .... - \pi_k$

Πy_{t-1} explains the stable long run relationship between the level of the variables and the remaining terms explains the short run changes. The dynamic properties of the model depend upon the properties of the Π matrix. In order to determine the number of cointegration relation we need to determine the rank of this matrix Π. If this matrix has full rank i.e. "p" then all variables in the model are stationary and there is no problem. If it has reduced rank "r" than there are some non-stationary variables in the model and there are "r" cointegration relations and "n-r" non stationary variables. "Π" with reduce rank can then be decomposed in the matrix of coefficients for explaining long run relation and the adjustment matrix i.e. $\Pi = \alpha\beta'$.

Where α = Adjustment matrix meaning how quickly the variables respond to correct for disequilibrium errors.

β' = matrix of long run relationship or matrix of cointegration relation.

Johansen (1988) has provided two procedures to test the rank of matrix Π. The null hypothesis of "r" cointegration relation (rank = r) is tested against the alternative hypothesis of greater number of relation using trace statistic which is given by the following formula.

$$\kappa = -T \sum_{i=r+1}^{p} \ln(1-\lambda_i) \quad \lambda_i = \text{ith eigen value}$$

In second procedure null hypothesis of rank r is tested against the alternative hypothesis of rank r +1 by using max statistics which is give as follows:

$$\kappa = -T \ln\left(1-\hat{\lambda}_{r+1}\right)$$

Where λ 's are estimated eigen values from estimated matrix Π.

5.3 Cointegration I (2) Analysis for Nominal Variables

First of all we did I(2) analysis in order to see if there are some I(2) trend in the nominal variables. The results are reported in table A.1 and A.2 in the appendix. Results show that there is possibility of I(2) trends hence we transformed our variables in real in order to avoid complications involved in the analysis of I(2) trends.

5.4 Modelling Demand for Narrow Money (M_1)

We estimated unrestricted VAR model with three variables real M_1, real GDP, and interest rate. The order of the VAR is fixed at 2 by using AIC criterion and LM test (AR-1) for detecting serial correlation. Although we are using real variables and we are not expecting I (2) trend but we will do I (2) analysis in real variables just to make sure that there is no I (2) trend in the data. The results of unrestricted I (2) analysis are reported in the appendix table A3 and results confirmed that there are no I (2) trends in the data.

Results for the I (1) cointegration analysis for testing rank using trace statistic are reported in table (5). The results show that there could be one cointegration relation at 10% level meaning one long run relationship between variables that can be interpreted as money demand relation. Error correction model is then estimated with the restriction of one cointegration relation. We have tested to restrict the constant in to the cointegration relation but LR test rejected the hypothesis of restricted constant hence constant can not be restricted to cointegration space. This is due to the fact that there is trend in the level of the series.

Table: 5 Cointegration analysis; Demand for Narrow Money M_1

H0: Rank	Trace test	p-value
0	28.484	[0.071]
1	4.6619	[0.840]
2	0.10429	[0.747]

The results of the stable long run relationship β and adjustment vector α are reported in table (6). The signs of the coefficients are as expected. GDP is positively related with money demand where as interest rate has negative relation with money demand. We have also tried to include inflation in the model to test whether it is playing any role in the determination of long run relationship but its coefficient turned out to be positive which is not according to theory so we have not included it in the model. The magnitude of the coefficients is showing that money demand is more sensitive to changes in GDP and less sensitive (less elastic) to the changes in interest rate. We tested the hypothesis to see the

Table: 6. Stable Long Run Money Demand Function for M_1

	Money Demand M_1	Log of Real GDP	Interest Rate
β	1.000	1.045 (0.034)	-0.0122 (0.008)
α	-0.588 (0.123)	0.029 (0.059)	5.276 (2.761)

Standard errors of the estimates are reported in the parenthesis

significance of interest rate for money demand in the long run and we accepted the hypothesis that interest is not affecting M_1. This result is quite obvious as we have estimated the money demand function for narrow definition of money which consist of most liquid form of money hence interest rate is not affecting demand for this money. People only demand the money for their daily transactions.

Adjustment coefficient for money demand M_1 is negative meaning that this variable responds to correct the disequilibrium errors of the pervious period. The adjustment coefficient for GDP and interest rate is positive meaning that these variables respond in opposite direction to correct for the disequilibrium errors but they seem to be insignificant. We tested the significance of adjustment coefficients for GDP and interest rate individually and simultaneously by using likelihood ratio test and both hypothesis are accepted. We can conclude that these two variables are found to exogenous in this long run relationship. We tested earlier that interest rate is not affecting money demand in the long run but again we test its joint significance with other accepted restriction and the results are reported in table 7. LR statistic has accepted this joint hypothesis at 5% level hence the resulting is more efficient long run relationship between money demand and GDP.

Table: 7. Testing the Joint Hypothesis for Exogenity of GDP and Interest Rate And Interest Inelasticity of Money Demand for M_1

	Money Demand M_1	**Log of Real GDP**	**Interest Rate**
β	1.000	0.985 (0.027)	0000 (0000)
α	-0.426 (0.094)	0000 (0000)	0000 (0000)
LR test of restrictions: Chi^2(3) = 6.3150 [0.0973]			

We can test one more interesting hypothesis that the coefficient of GDP equal to one. The interpretation of this hypothesis is that price will be cancelled in this relationship and there will be no difference between the model with nominal variables and model with real variables. This hypothesis also corresponds to traditional quantity theory of money. The results of the joint test with other accepted restriction are reported in table 8 and LR test rejected the hypothesis.

Table: 8. Testing the hypothesis for coefficient of GDP equal to one

	Money Demand M_1	Log of Real GDP	Interest Rate
β	1.000	1.000 (0000)	0000 (0000)
α	-0.0046 (0.094)	0000 (0000)	0000 (0000)
LR test of restrictions: Chi^2(4) = 23.372 [0.0001]			

5.6 Modelling demand for Broad Money M_2

We already mentioned that in modelling nominal variables that there is evidence of I (2) trends hence we transformed our variables in real to avoid complications. In order to make sure that there are no I (2) trends in real variables we did I (2) cointegration analysis with real variables. The results of the analysis are reported in appendix A2. These results suggest that there is no evidence of I (2) trends in real variables.

The lags of the unrestricted VAR model are fixed at 2 using AIC and LM test (AR-1) for detecting serial correlation. The results of I (1) cointegration analysis are reported in table (9) using trace statistics. The results show that there is one cointegration relation between these variables that can be interpreted as money demand relation. We then estimated the error correction model to find the coefficients of long run money demand relation. First in error correction formation we tested whether constant can be restricted to cointegration space or not. The hypothesis is rejected so we can not restrict the constant to be in the cointegration space have the interpretation that there is trend in the level of the series.

The results of stable long run relation with unrestricted constant are reported in table (10). The sign of the coefficient for GDP is positive as suggested by theory. The interest rate coefficient is also positive which is not according to theory but it is statistically insignificant. We applied likelihood ratio test to test the significance of the coefficient of interest and hypothesis is accepted at 5% level of significance. This result is not surprising for an underdeveloped country like Pakistan since most of the people are very poor and living on the subsistence level. They demand money only to fulfil their daily transaction and they don't have long run perspective of holding money. There could be another reason that financial markets are imperfect and there is high inflation rate.

Table: 9. I (1) Cointegration analysis Demand for Broad money M_2

H0:rank	Trace test	p-value
0	33.325	[0.018]
1	4.7219	[0.834]
2	0.055926	[0.813]

Table: 10. Stable Long Run Money Demand Function for M_2

	Money Demand M_2	Log of Real GDP	Interest Rate
β	1.000	1.109 (0.033)	0.013 (0.007)
α	-0.539 (0.098)	-0.027 (0.053)	2.554 (2.551)

Standard errors of the estimates are reported in the parenthesis

The adjustment coefficients for money demand M_2 and GDP are negative meaning that these variables adjust to correct for past disequilibrium but the coefficient for interest is positive. The significance of these coefficients is tested using likelihood ratio test and results suggested that the coefficient of adjustment for both GDP and interest rate turn out to be statistically insignificant individually and simultaneously. These results suggest that these two variables are exogenous in the long run money demand relation.

We then tested the joint hypothesis that money demand is inelastic with respect to interest rate and GDP and interest are exogenous in the long run money demand relation. The hypothesis is accepted and the final stable money demand relation is reported in table (11). We again tested the hypothesis for coefficient of GDP to be equal to one but again we rejected that hypothesis as in the case of modelling M_1.

5.7. Estimation of model in Sub Samples.

We have seen in the univariate analysis that there is structural break in the series so it is very important to incorporate that in the multivariate analysis but it is bit complicated to include dummy variables in the VAR models. Instead we

Table: 11. Testing the Joint Hypothesis for Exogenity of GDP and Interest Rate and Interest Inelasticity of Money Demand for M_2

	Money Demand M_2	Log of Real GDP	Interest Rate
β	1.000	1.146 (0.024)	0000 (0000)
α	-0.451 (0.083)	0000 (0000)	0000 (0000)
LR test of restrictions: Chi^2(3) = 3.9753 [0.2641]			

Standard errors of the estimates are reported in the parenthesis.

have estimated our model in the sub sample that is sample before the break and sample after the break to see if there is some change in the results. The results are reported in table A5 and A6 for both M_1 and M_2 respectively. Results have shown that there is no cointegration relation for both M_1 and M_2 before the break (1953- 1973) and there is one cointegration relation for sample after the break (1974-2003). For the sample after the break we found almost same results as we obtained for the whole sample.

VI. Conclusion

In this study we have tried to estimate stable money demand function for Pakistan using 51 annual observations from 1953 to 2003. The results of unit root analysis have suggested that both log of nominal GDP and price could possibly be integrated of order (2). The results unit root analysis for real M_1 and M_2 by allowing structural break in the series, due to oil price shock of the history, have suggested that M_1 is fractionally integrated where as M_2 is trend stationary by using both ADF and KPSS test.

I(2) cointegration analysis with nominal variables have suggested there are some I(2) trends in the model hence in order to avoid the complications involved in I(2) analysis we transformed our model in real variables. We tested again for I

(2) trends in the model with real variables just to make sure that there are no I (2) trends in the model and we found no evidence for I (2) trends.

We found one cointegration relation for both definitions of money. Signs of the coefficients are according to theory for M_1 money demand relation. For M_2 interest rate has wrong sign but it is statistically insignificant. Both GDP and interest rate are found to be weakly exogenous in the long run money demand relation. We also found that demand for both M_1 and M_2 are inelastic with respect to interest rate. This finding is quite obvious for an underdeveloped country like Pakistan where most of the people are living on subsistence level.

The main conclusion from our study is that demand for both types of real money does not respond to the changes in the interest rate but they do respond to the changes in real GDP. The implication of this finding could be that the monetary authorities can not use interest rate as a policy variable to adjust money demand.

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Appendix

A.1 Graph of the Log of Nominal Series

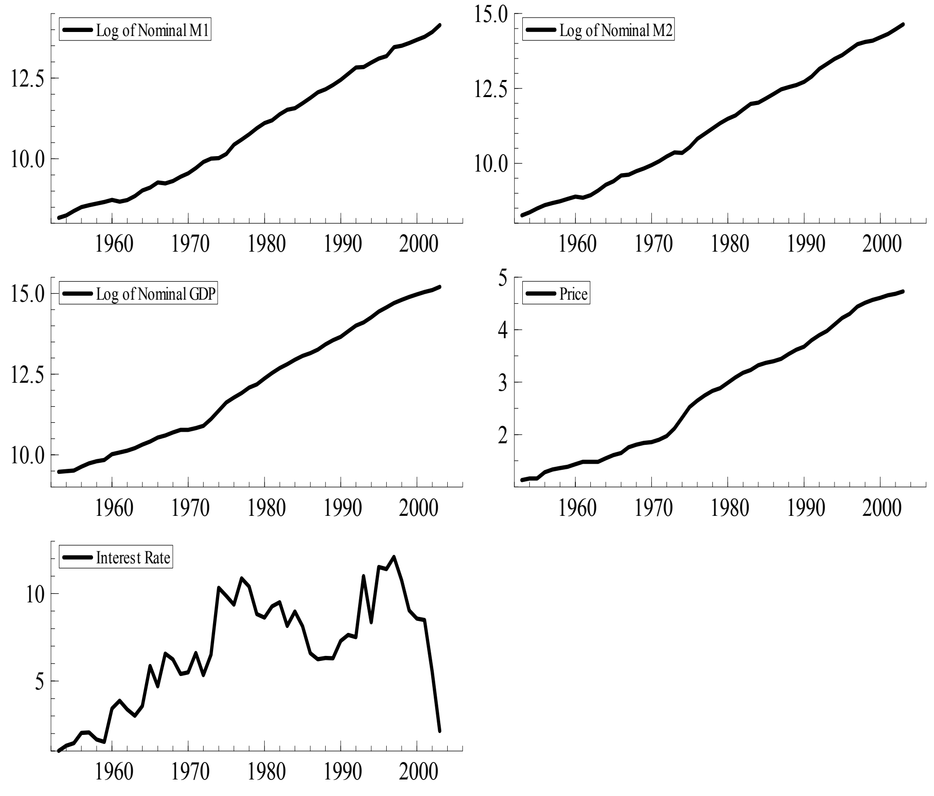

Figure A2. Graph of the Log of real series.

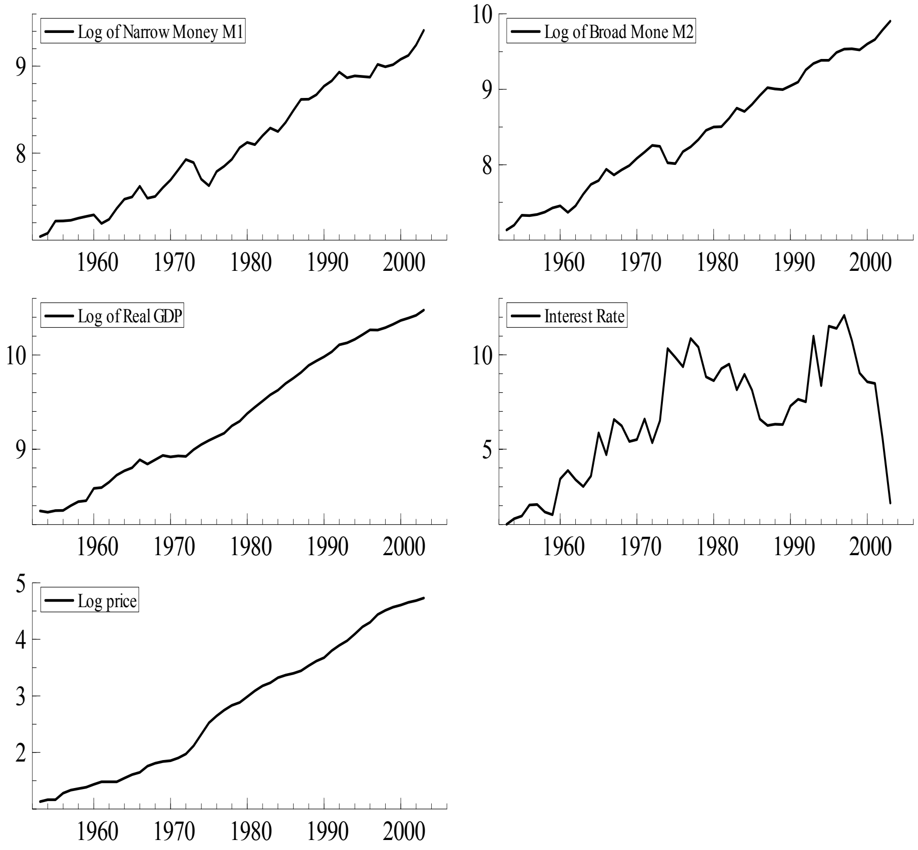

Table A.1 Testing for I (2) Trends in Nominal Variables
Modelling: Nominal M1, Log of Nominal GDP,
Interest Rate and Log Prices

p=	0	1	2	3
n-p-s=0	0.4938	0.345	0.1394	0.0017
Q_p	61.594	28.25	7.4423	0.086
[pval]	**0.0012**	**0.0761**	**0.5337**	**0.7689**
n-p-s=	4	3	2	1
p = 0	0.60697	0.578	0.4018	0.205
S_p,s	186.14	140.38	98.044	72.86
[pval]	**0.0000**	**0.0000**	**0.0001**	**0.0016**
p = 1	0	0.5867	0.5441	0.2401
S_p,s	0	123.5	80.189	41.692
[pval]	**0.0000**	**0.0000**	**0.0000**	**0.0232**
p = 2	0	0	0.53443	0.1434
S_p,s	0	0	52.489	15.02
[pval]	**0.0000**	**0.0000**	**0.0005**	**0.3952**
p = 3	0	0	0	0.342
S_p,s	0	0	0	20.649
[pval]	**0.0000**	**0.0000**	**0.0000**	**0.0060**

Table A.2 Testing for I (2) in Nominal Variables
Modelling Nominal M2, Log of Nominal GDP,
Interest Rate and Log Prices

p=	0	1	2	3
n-p-s=0	0.476	0.358	0.118	8.94e-00
Q_p	59.598	27.884	6.1682	0.0043
[pval]	**0.0022**	**0.083**	**0.679**	**0.947**
n-p-s=	**4**	**3**	**2**	**1**
p = 0	0.594	0.541	0.355	0.156
S_p,s	171.91	127.71	89.475	67.962
[pval]	**0.000**	**0.000**	**0.0015**	**0.0059**
p = 1	0	0.585	0.394	0.158
S_p,s	0	104.04	60.902	36.356
[pval]	**0.000**	**0.000**	**0.005**	**0.090**
p = 2	0	0	0.421	0.155
S_p,s	0	0	41.615	14.47
[pval]	**0.000**	**0.000**	**0.0151**	**0.441**
p = 3	0	0	0	0.280
S_p,s	0	0	0	16.133
[pval]	**0.000**	**0.00**	**0.00**	**0.026**

A. 3 I (2) Cointegration analysis on Real Variables Modelling: Real M1, Real GDP, and Interest Rate.

p=	0	1	2
n-p-s=0	0.38501	0.088819	0.002126
Q_p	28.484	4.6619	0.10429
[pval]	**0.0713**	**0.8401**	**0.7467**
n-p-s=	**3**	**2**	**1**
p = 0	0.59846	0.57826	0.36486
S_p,s	137.74	93.03	50.725
[pval]	**0.0000**	**0.0000**	**0.0014**
p = 1	0	0.58933	0.55426
S_p,s	0	87.864	44.255
[pval]	**0.0000**	**0.0000**	**0.0000**
p = 2	0	0	0.53655
S_p,s	0	0	37.788
[pval]	**0.0000**	**0.0000**	**0.0000**

A.4 I (2) Cointegration analysis on Real Variables Modelling: Real M2, Real GDP, and Interest Rate.

p=	0	1	2
n-p-s=0	0.44218	0.0908	0.0011
Q_p	33.325	4.721	0.0559
[pval]	**0.0180**	**0.834**	**0.813**
n-p-s=	**3**	**2**	**1**
p = 0	0.58911	0.5051	0.353
S_p,s	132.77	89.185	54.717
[pval]	**0.000**	**0.000**	**0.0004**
p = 1	0	0.56456	0.412
S_p,s	0	71.486	30.747
[pval]	**0.000**	**0.000**	**0.002**
p = 2	0	0	0.501
S_p,s	0	0	34.197
[pval]	**0.000**	**0.000**	**0.000**

Table A.5 Estimation Results for Sub Samples: Long Run Relationship before and After the Break for M1

Variable	Results of Sub Sample 1953-1973 **Pre Break Sample**	Estimates of Sub Sample 1953-1973 **After Break Sample**		Estimate of the **Full Sample** 1974 – 2003	
		α	β	α	β
Money Demand M1	No Long Run relationship found	-0.750 (0.111)	1.000	-0.426 (0.094)	1.000
Log of Real GDP		0000 (0000)	1.1347 (0.026)	0000 (0000)	0.985 (0.027)
Interest Rate		0000 (0000)	0000 (0000)	0000 (0000)	0000 (0000)
Test of Valid Restrictions Imposed		LR test of restrictions: Chi^2(3)=3.5276 **p-value [0.3172]**		LR test of restrictions: Chi^2(3)=6.3150 **P-value [0.0973]**	

Table A.6 Estimation Results for Sub Samples: Long Run Relationship before and After the Break for M2

Variables	Results of Sub Sample 1953-1973 **Pre Break Sample**	Estimates of Sub Sample 1953-1973 **After Break Sample**		Estimate of the **Full Sample** 1974 – 2003	
		α	β	α	β
Money Demand M2	No Long Run relationship found	-0.858 (0.157)	1.000	-0.451 (0.083)	1.000
Log of Real GDP		0000 (0000)	1.032 (0.025)	0000 (0000)	1.146 (0.024)
Interest Rate		0000 (0000)	-0.022 (0.007)	0000 (0000)	0000 (0000)
Test of Valid Restrictions Imposed		LR test of restrictions: Chi^2(2)=5.3262 **p-value [0.0697]**		LR test of restrictions: Chi^2(3)=3.9753 **p-value [0.2641]**	