

Estimating Food Demand Elasticities in Pakistan: An Application of Almost Ideal Demand System

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Abstract

The main focus of the study is to estimate the rural-urban income and own price elasticities across a range of consumption quintiles. The Linear Approximate Almost Ideal Demand System (LAAIDS) is used to estimate the parameters of aggregate food commodity groups. Due to the specific features of the data, spatial variations in regional prices are estimated and used as proxies for food prices (i.e. unit values) by using household survey data. Regarding household specific elasticity estimates, households exhibit increasing consumption of vegetables, fruits, milk and meats with higher income. The expenditure elasticities are larger in rural areas compared to urban areas and expenditures on most food groups increase at a decreasing rate as income increases. Expenditure elasticities for all food groups were positive and less than one, except for fruits, meats, and milk that have been identified as luxuries. Cereals tend to have the lowest expenditure elasticity of demand. The uncompensated own-price elasticities of demand for all food groups are negative and their absolute amounts are lower than unity i.e. demand reacts in-elastically to own-price changes, except for meats (elastic). According to the values of the cross-price elasticities and on the level of all selected food groups, only substitution relationships are observed. The high price elasticities of demand for many food items stress the importance of food price changes for households, and their reactions should be taken into account in the development of comprehensive agricultural and food policies in Pakistan.

Keywords: Consumer demand analysis; PIHS data; LAAIDS; price and expenditure elasticities

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I. Introduction

Demand elasticities for a particular country provide valuable information for policy analysts in understanding the pattern of growth of the national food consumption. Specific country elasticities are influenced by both the level of income attained and the quantities of food that are currently eaten by the consumer. Estimation of complete demand functions is incredibly useful not only in obtaining price elasticities, but also in getting reliable estimates of expenditure (income) elasticities. The measurement of these elasticities is required for the design of many different policies; for example, intelligent policy design for indirect taxation and subsidies requires knowledge of these elasticities for taxable commodities and, in addition, in the projections for future food consumption².

Such knowledge would normally be obtained by the analysis of time-series data on demand for commodities, prices, and incomes. For Pakistan as well as for many developing countries, there is typically rather few time-series data from which price elasticities can be inferred. As a result of this limitation and with the available cross-sectional data resulting from extensive surveys on household expenditures, most studies in Pakistan concentrated on the estimation of expenditure elasticities (Engel relationship) and overlooked the price elasticities.

As the estimation of complete demand functions is incredibly useful not only in obtaining price elasticities, but also in getting reliable estimates of expenditure (income) elasticities, so towards this end the study lays out the estimated rural-urban income and own price elasticities, across a range of consumption quintiles, of aggregated food groups. Section II addresses the issue that how price elasticities could be estimated from cross sectional data? Section III is specified for model specification of LAAIDS, for the estimation of complete demand system along with the description of income and price elasticity formulas. Section IV highlights the adopted estimation technique along with description of the variables. The empirical findings are reported in section V. Concluding remarks are presented in section VI along with policy implications for Pakistan.

² See e.g. Deaton (1986, 1987, 1988), for a meticulous discussion.

II. Price Elasticities from HIES data

Deaton (1987) developed a methodology by using household survey data to detect the spatial variation in prices and to estimate the price elasticities by comparing spatial price variation to spatial demand patterns. The household surveys contain information on the spatial distribution of prices, and thus, by recovering this information in a useful form, there is a potential for estimating the impact of prices on quantity demanded. Since prices for food products are not provided by the survey, the ratio of expenditure to purchased quantity can be used as a proxy for prices. These prices should be corrected before being incorporated into the demand system according to the causes of cross-sectional price variations.

Prais and Houthakker (1955, 1971) identify price variation due to region, price discrimination, services purchased with the commodity, seasonal effects, and quality differences caused by heterogeneous commodity aggregates. When the structure of demand is relatively constant, price variation can be attributed to changed supply conditions and can be used to identify commodity demand curves. In order to interpret correctly the effects of prices in the analysis of household budget data, the causes of cross-sectional price variations must be identified and only supply related price variations should be used to estimate the demand functions.

In the survey data used by Deaton, there are variations in the cross-sectional price data due to region, household characteristics (male, female, age groups), seasonal effects, aggregation of the commodities, etc. Similar data for the survey data used by Deaton are available for a wide range of developing countries so that the technique should have wide applicability.

Keeping in line with the methodology of Deaton nine aggregated food commodity groups were chosen for the analysis of this study: cereals (CR), pulses (PL), fruits (FR), edible oils and fats (EOF), sugar and gur (SG), meats (MT), vegetables (VG), tea, coffee and soft drinks (TCS), and milk and milk products (MMP). Each of selected food group is not a homogeneous good but consists of a number of components. For example, in the data it is possible to separate the cereals group into wheat, rice, and maize, but a category such as "rice" does not encompass different kinds of rice, some of which are more expensive than others. This food-grouping is to reduce the total number of parameters in the model and then estimation demand system more

manageable³. Each food group includes those commodities that have the same nutritional value and their prices are very likely to move in tandem and hence there would be no serious aggregation problem.

The variation in food group prices is due to differences in consumed items in each group and the variation in prices of each item across provinces. The latter is due to regional market conditions. Therefore, the price of each food group is computed as a weighted average of prices on specific items. The price obtained is effectively a value and quantity ratio, which is called a “unit value” by Deaton (1988) and consequently could be used as a proxy of prices. This “unit value” as defined by Deaton is used in this study after the name of “unit value of the aggregated commodity”.

Using unit values as price proxies, as in this study, brings about another specific concern. Unit values are not only affected by the actual prices consumers face, but also by the composition of the commodity group. When separate goods are aggregated into a single commodity group, this leads to variations in the average price, *i.e.* unit value of the aggregated commodity, changing with the quantities of the goods of which it is composed. This means that quality choice in this context is not only a question of differentiated goods but also quality choice is reflected in the quantity shares of the component goods.

The published data of the Pakistan’s HIES is aggregated at eight rural-urban regions across four provinces. The ratio of expenditure to quantity, the cost of the purchase, gives the cost of the commodities for four provinces. This information can be used as a proxy for the prices after calculating the “unit value of the aggregated commodity”. Given, for example, different cereals costs, and then, there will be spatial variation in the costs of this food group across the regions. This variation can be used to obtain the price information, which is missing in the household survey data. Thus, a complete demand system can be estimated, and price and expenditure (income) elasticities can be calculated as a result.

So, in continuation of the previous discussion and keeping in mind the specific features of the data, the study has made use of spatial variation in regional prices estimated using household survey data. The estimated spatial variation in regional prices, as per methodology suggested by Deaton, is used as proxies for food prices. They are incorporated into the complete food

³ See for instance Abdulai (2002, 2003); and Abdulai and Aubert (2004).

demand analysis, *i.e.* LAAIDS after calculating the “unit value of the aggregated commodity”, to measure own and cross price elasticities for an assortment of food groups.

III. The LAAIDS

The LAAIDS has been chosen as the basic model for the complete demand system estimation in this study due to its flexible functional form and nimbleness in estimation. In a short and snappy way the demand function of LAAIDS in budget share form can be expressed as:

$$w_{ic} = \alpha_i + \sum_j \gamma_{ij} \ln p_{jc} + \beta_i \ln px_c / P_c^* \quad (1)$$

Where the commodities $i = 1, \dots, 9$ and the consumption quintile $c = 1, \dots, 5$. w_{ic} is the budget share of good i in the respective consumption quintile c , p_{jc} is the price of good j in the respective quintile, x_c is household's total food expenditure in the specific quintile c . P_c^* is the stone's price index, and α_i , β_i , and γ_{ij} are the parameters that need to be estimated.

The demand elasticities are calculated as functions of the estimated parameters, and they have standard implications. The specific form of expenditure elasticity (ε_i), which measures sensitivity of demand in response to changes in consumption expenditure, is as:

$$\varepsilon_i = 1 + (\beta_i / w_i) \quad (2)$$

The uncompensated (Marshallian) own-price elasticity (ε_{ii}) and cross-price elasticity (ε_{ij}) measure how a change in the price of one product affects the demand of this product and other products with the total expenditure and other prices held constant. The specific form of uncompensated own and cross price elasticities is as, respectively:

$$\varepsilon_{ii} = (\gamma_{ii} / w_i) - (\beta_i + 1) \quad (3)$$

$$\varepsilon_{ij} = (\gamma_{ij} / w_i) - (\beta_i w_j / w_i) \quad (4)$$

The compensated (Hicksian) price elasticities own and cross (ε_{ii}^* and ε_{ij}^*), which measures the price effects on the demand assuming the real expenditure x_c / P_c^* is constant, is described as:

$$\varepsilon_{ii}^* = (\gamma_{ii} / w_i) + w_i - 1 \quad (5)$$

$$\varepsilon_{ij}^* = (\gamma_{ij}/w_i) + w_j \quad (6)$$

Also, the compensated price elasticity can be derived easily by using ε_i , ε_{ii} , and ε_{ij} , and the following relation:

$$\varepsilon_{ij}^* = \varepsilon_{ij} + \varepsilon_i \times w_j \quad (7)$$

In particular, the sign of the calculated ε_{ij}^* indicates the substitutability or complementarity between the destinations under consideration.

Using the LAAIDS model to estimate the two-stage budgeting demand function presents several advantages. Probably the most important is that it is a flexible functional form. The LAAIDS substitution pattern implies an unconstrained pattern of conditional cross-price across products within sub-segments. This is an advantage, because competition is probably higher among differentiated products within sub-groups. Another important advantage of the LAAIDS model is the perfect aggregation over consumers, without requiring linear Engle curves. This is very important in studies of aggregate data. Finally, the demand function derived from this model crosses the price axis, avoiding the presence of virtual prices.

IV. Data and Estimation Procedure

Data for this study is obtained from the Federal Bureau of Statistics (FBS) for the year of 2007-08. FBS provided an electronic copy of the data sets for four provinces aggregated into five consumption percentiles. The cost indices of the bundles of the aggregated food commodities are calculated from the given data set. The expenditure data are pooled across the four provinces and five consumption percentiles in each province in the study. It is assumed that cost indices of the bundles of the food commodities are only different across the provinces and for each consumption quintile, but not within the province according to Deaton's methodology. In simple words it is assumed that households at different consumption percentiles have the same cost indices for the aggregated food commodities within the same province. The cost indices of these commodities in each Province are used as proxies for prices and hence enabled us to estimate income and price elasticities across these defined consumption quintiles.

No regional elasticities (rural and urban) are estimated keeping in line with the assumption of no variation in the unit values within the same region. Our study includes nine aggregated food commodity groups, as defined

earlier. The prices for these commodity aggregates will be proxied by the cost of these commodity aggregates in each province across the quintiles.

A system of share equations based on first equation and subject to the restrictions (adding-up, homogeneity, and symmetry) is estimated using Iterative Seemingly Unrelated Regression (ISUR) method of Zellner. This method is equivalent to Full Information Maximum Likelihood (FIML) estimation. The adding-up property of demand causes the error covariance matrix of system to be singular, so one of the expenditure share equations is dropped from the system to avoid singularity problems. The estimates are invariant of which equation is deleted from the system. Homogeneity is maintained by normalizing all of the prices (proxied by the aggregate cost figures) by the price of others group (OT). The coefficients pertaining to the expenditure share equation of others aggregate (OT), which is dropped from the system in the estimation stage, are obtained by using the adding-up property. Symmetry is imposed during the estimation of the system of equations. Now, we present the results of our estimation. The above models are initially estimated for the whole sample of households, regardless of their income and consumption levels. Later, households are split according to consumption quintiles, and the models are estimated for each group.

V. Model Results

The above model in first equation was initially estimated for the whole sample of households, regardless of their respective consumption quintiles. Later, households were split according to their consumption patterns, and the models were estimated for each group. Following Green and Alston (1990, 1991), we assume that the preference structure is such that, in the first stage, consumers choose how to spend their income among groups of products, such as food, housing, transportation, health services, education, etc. In the second stage, the level of expenditure in each group, as determined in the first stage, is allocated to the commodities in that group.

The empirical results for the specified model for demand functions (LAAIDS) illustrate that all estimated coefficients agree with a priori theoretical expectations. As a result of 2nd stage of the two-stage budgeting process the estimates of the structural parameters for food groups of the LAAIDS model for the whole sample of households are shown in Table 1. Following the same line of action, the parameters of LAAIDS for 1st quintiles (low income households) and 5th quintiles (high income households) quintiles are reported in Table 2 and 3 respectively. The equation for milk and milk

products was excluded to avoid singularity, but its coefficients were later recovered with the use of the homogeneity property. The parameters estimates satisfy the adding-up restriction. Overall, it can also be seen from the estimated results that a reasonable number of coefficients of the explanatory variables are significant. Out of eighty one coefficients we have twenty five γ_{ij} 's with significant t-statistics.

However of interest to researchers and policy makers is the knowledge concerning elasticities of demand for food. According to value of the expenditure elasticities, the selected food groups are classified as inferior goods ($\epsilon_i < 0$), necessities ($0 < \epsilon_i < 1$), or luxuries ($\epsilon_i > 1$). Demand for a specific commodity is defined as price inelastic (elastic), if the absolute value of its own-price elasticity is lower than unity (larger than unity).

Pairs of commodities are denoted as substitutes or complements if their compensated cross-price elasticities are positive or negative, respectively. Compensated elasticities indicate the change in demand for a commodity due to a price variation, when the real expenditure change caused by this price variation is compensated by an expenditure variation so that utility is kept constant. Using formulae given in equation (2) to (6) the expenditure, uncompensated and compensated price elasticities, respectively, are presented in Tables 4 through 12. The calculated elasticities and the relative order of magnitude among them are reasonable as compared with those values one would expect given heuristic considerations.

5.1. Expenditure elasticities

Table 4 displays the expenditure consumption (income) and own-price elasticities for the food sub groups for the whole sample. Generally, the expenditure elasticities for selected food groups in Pakistan are relatively high. This can be explained by the economic situation in Pakistan. Many households, especially the poor, face tight budgetary constraints and all of the selected food commodity groups are considered as very important items because they fulfill fundamental needs of people.

It can be seen from the Table 4 that expenditure and own-price elasticities are of expected sign. The income (expenditure) elasticities for all food groups are positive and less than one ($0 < \epsilon_i < 1$), except for fruits, meats, and milk and milk products, indicating that food groups are normal and necessary goods, and there are no inferior products. For pulses, the expenditure elasticity

Table: 1. Parameter Estimates of LAAIDS for Total Sample and for Aggregated Food Groups

Food Groups	α_i	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}	γ_{i8}	γ_{i9}	R^2
Cereal (CR)	0.423	-0.078	0.060	0.103	-0.178	0.194	-0.201	0.087	-0.102	-0.063	-0.030	0.798
	1.419	-1.709*	0.128	1.227	-2.134**	1.698*	-2.957**	1.065	-2.745**	-0.535	-1.046	
Pulses (PL)	0.048	-0.015	0.003	0.084	0.023	0.006	-0.069	0.006	-0.010	0.102	0.007	0.820
	0.507	-1.000	0.254	3.132***	0.879	0.152	-3.230***	0.238	0.900	2.714	0.755	
Fruits (FR)	-0.039	0.009	-0.005	-0.020	0.008	0.066	-0.020	-0.015	0.001	0.006	0.011	0.793
	-0.987	1.603*	-0.743	-1.733*	0.699	4.256***	-2.257**	-1.414	0.046	0.417	3.055***	
Edible Oil & Fats (EOF)	-0.066	-0.007	0.020	0.011	-0.020	0.029	-0.007	-0.002	0.001	0.070	0.005	0.839
	-1.269	-0.895	2.441**	0.806	-1.358	1.478	0.565	-0.163	0.029	3.378***	0.862	
Sugar (SG)	0.235	-0.022	-0.017	0.023	-0.003	-0.032	0.020	-0.002	0.013	-0.067	0.006	0.802
	3.392***	-1.770*	-1.490	1.048	-0.190	-1.096	1.090	-0.143	1.259	-2.120**	0.799	
Meats (MT)	0.176	0.099	-0.045	-0.069	0.086	-0.023	0.041	0.015	0.029	-0.089	0.008	0.789
	0.586	2.170**	-0.970	-0.801	1.029	-0.207	0.598	0.177	0.760	-0.742	0.285	
Vegetables (VG)	0.091	-0.011	0.036	0.037	-0.043	-0.154	0.120	-0.060	0.022	0.162	-0.023	0.756
	0.478	-0.385	1.187	0.686	-0.799	-2.094**	2.724	-1.145	0.913	2.110	-1.206	
Tea, Coffee & Soft Drinks (TCS)	0.132	-0.008	0.002	0.006	0.045	0.036	-0.026	-0.032	-0.001	0.007	-0.009	0.812
	2.487**	0.915	0.282	0.412	2.990**	1.700*	-2.193**	-2.207**	-0.063	0.327	-1.877*	
Milk & Milk Products (MMP)	0.149	0.033	-0.056	-0.170	0.074	-0.121	0.141	-0.002	0.060	-0.129	0.022	0.836
	0.941	1.399	-2.315**	-3.774***	1.660	-1.987*	3.912***	-0.057	3.015***	-2.020**	1.380	

Note: 2nd line of each group describes the t-values, in smaller font size. * * * Indicates significant at one percent level of significance, * * Indicates significant at five percent level of significance and * Indicates significant at ten percent level of significance.

Table: 2. Parameter Estimates of LAAIDS for Quintile 1st and for Aggregated Food Groups

Food Groups	α_i	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}	γ_{i8}	γ_{i9}	R^2
Cereal (CR)	0.396	-0.073	0.056	0.097	-0.167	0.182	-0.188	0.082	-0.096	-0.059	-0.028	0.789
	1.326	-1.597*	0.119	1.147	-1.995*	1.588*	-2.765**	0.995	-2.566**	-0.500	-0.978	
Pulses (PL)	0.045	-0.014	0.003	0.078	0.021	0.005	-0.064	0.005	-0.010	0.096	0.006	0.845
	0.474	-0.935	0.238	2.928**	0.821	0.142	-3.020***	0.223	0.842	2.537**8	0.706	
Fruits (FR)	-0.037	0.009	-0.004	-0.018	0.008	0.061	-0.018	-0.014	0.001	0.005	0.011	0.812
	-0.922	1.498	-0.694	-1.620*	0.654	3.978***	-2.110**	-1.322	0.043	0.390	2.856**	
Edible Oil & Fats (EOF)	-0.061	-0.006	0.018	0.011	-0.018	0.027	-0.006	-0.002	0.001	0.066	0.004	0.862
	-1.187	-0.836	2.282**	0.754	-1.270	1.381	0.528	-0.153	0.027	3.158***	0.806	
Sugar (SG)	0.219	-0.020	-0.016	0.021	-0.003	-0.030	0.018	-0.002	0.012	-0.062	0.005	0.798
	3.171***	-1.654*	-1.393	0.979	-0.177	-1.024	1.019	-0.133	1.177	-1.982*	0.747	
Meats (MT)	0.164	0.092	-0.042	-0.064	0.081	-0.021	0.039	0.014	0.027	-0.083	0.008	0.789
	0.548	2.028**	-0.907	-0.749	0.962	-0.193	0.559	0.166	0.711	-0.693	0.267	
Vegetables (VG)	0.085	-0.011	0.033	0.034	-0.040	-0.144	0.112	-0.056	0.020	0.152	-0.021	0.776
	0.447	-0.360	1.109	0.642	-0.747	-1.957*	2.547**	-1.071	0.854	1.973*	-1.128	
Tea, Coffee & Soft Drinks (TCS)	0.124	-0.008	0.002	0.005	0.042	0.033	-0.025	-0.030	-0.001	0.006	-0.009	0.750
	2.325**	0.856	0.263	0.385	2.795**	1.589*	-2.050**	-2.063**	-0.059	0.305	-1.754*	
Milk & Milk Products (MMP)	0.140	0.031	-0.053	-0.159	0.069	-0.113	0.132	-0.002	0.056	-0.120	0.020	0.861
	0.879	1.308	-2.16**	-3.5***	1.552*	-1.857*	3.657***	-0.054	2.818**	-1.889*	1.290	

Note: 2nd line of each group describes the t-values, in smaller font size. * * * Indicates significant at one percent level of significance, * * Indicates significant at five percent level of significance and * Indicates significant at ten percent level of significance.

Table: 3. Parameter Estimates of LAAIDS for Quintile 5th and for Aggregated Food Groups

Food Groups	α_i	β_i	γ_{i1}	γ_{i2}	γ_{i3}	γ_{i4}	γ_{i5}	γ_{i6}	γ_{i7}	γ_{i8}	γ_{i9}	R^2
Cereal (CR)	0.444	-0.082	0.063	0.109	-0.187	0.204	-0.211	0.092	-0.107	-0.066	-0.031	0.876
	1.490	-1.795*	0.134	1.289	-2.241**	1.784*	-3.106***	1.118	-2.883**	-0.562	-1.099	
Pulses (PL)	0.051	-0.016	0.004	0.088	0.024	0.006	-0.072	0.006	-0.011	0.107	0.007	0.875
	0.533	-1.051	0.267	3.290***	0.923	0.159	-3.392***	0.250	0.946	2.850**	0.793	
Fruits (FR)	-0.041	0.010	-0.005	-0.021	0.008	0.069	-0.021	-0.016	0.001	0.006	0.012	0.773
	-1.036	1.683*	-0.780	-1.820*	0.734	4.470***	-2.371**	-1.485	0.048	0.438	3.209***	
Edible Oil & Fats (EOF)	-0.069	-0.007	0.021	0.012	-0.021	0.030	-0.007	-0.002	0.001	0.074	0.005	0.824
	-1.333	-0.940	2.564**	0.847	-1.426	1.552*	0.593	-0.171	0.030	3.548***	0.906	
Sugar (SG)	0.246	-0.023	-0.018	0.024	-0.004	-0.034	0.021	-0.002	0.013	-0.070	0.006	0.817
	3.563***	-1.859*	-1.565*	1.100	-0.199	-1.151	1.145	-0.150	1.322	-2.227**	0.839	
Meats (MT)	0.185	0.104	-0.047	-0.072	0.091	-0.024	0.043	0.016	0.030	-0.093	0.008	0.797
	0.616	2.279**	-1.019	-0.842	1.081	-0.217	0.628	0.186	0.798	-0.779	0.299	
Vegetables (VG)	0.095	-0.012	0.037	0.039	-0.045	-0.162	0.126	-0.063	0.023	0.170	-0.024	0.740
	0.502	-0.405	1.246	0.721	-0.839	-2.199**	2.861**	-1.203	0.959	2.216**	-1.267	
Tea, Coffee & Soft Drinks (TCS)	0.139	-0.008	0.002	0.006	0.047	0.037	-0.028	-0.034	-0.001	0.007	-0.010	0.744
	2.612**	0.961	0.296	0.432	3.140***	1.785*	-2.303**	-2.317**	-0.066	0.343	-1.971*	
Milk & Milk Products (MMP)	0.157	0.035	-0.059	-0.179	0.077	-0.127	0.149	-0.002	0.063	-0.135	0.023	0.864
	0.988	1.470	-2.431**	-3.963***	1.744*	-2.087**	4.108***	-0.060	3.166***	-2.122**	1.449	

Note: 2nd line of each group describes the t-values, in smaller font size. * * * Indicates significant at one percent level of significance, * * Indicates significant at five percent level of significance and * Indicates significant at ten percent level of significance.

amounts to 0.871 and for vegetables and sugar and gur it amounts to 0.764 and 0.664, respectively. The food groups such as fruits, meats, and milk and its products have expenditure elasticities larger than unity ($\epsilon_i > 1$) which identifies them as luxuries. It is expected that these food groups will experience an increase in demand when consumers' income increases in tandem with the overall economic growth of the country. However, if real income of households further decreases, in relative terms, less expenditures will be allocated to these food commodities. This result indicates that as households' expenditures increase and households' diversify their diets, they tend to increase their consumption of non-staple foods rather than staple foods.

Table: 4. Expenditure (Income) and Marshallian Own-price Elasticities for Total Sample

Food group	Expenditure	Own-price
Cereals	0.541	-0.582
Pulses	0.871	-0.238
Fruit	1.327	-0.745
Edible oils and fats	0.821	-0.247
Sugar and gur	0.664	-0.672
Meats	1.222	-1.053
Vegetables	0.764	-0.290
Tea, coffee and soft drinks	0.833	-0.839
Milk and milk products	1.209	-0.898

Another interesting finding is that cereals tend to have the lowest expenditure elasticity of demand. The consumption of this group is relatively little affected by income changes and has already occupied a special position in the Pakistani's diet, as it is a staple food among the population.

The LAAIDS model permits the calculation of elasticities for different consumption quintiles, so in addition, expenditure elasticities has also been surged out for the poor and rich households of Pakistan (*i.e.* for 1st and 5th quintile) 1st quintile refers to the poor group and 5th quintile is meant for the upper class having high rate of consumption expenditure share. It is observed that income elasticities for almost all of the included groups are higher for lower class and lower for the rich class. It's as per the theoretical consideration that income elasticities move down ward as income increases and vice versa. So for poor high income elasticity is expected and the results of Table 5 confirm it. Among the food groups fruits; meats; tea, coffee and soft drinks; and milk and milk products with elasticities greater then one

seems to have a luxurious nature for the poor. In addition to these groups pulses; edible oils and fats; and vegetables with the expenditure elasticity close to one also conforming their existence very close to the luxurious items.

Table: 5. Expenditure (Income) and Marshallian Own-price Elasticities for 1st Quintile

Food group	Expenditure	Own-price
Cereals	0.653	-0.694
Pulses	0.878	-0.245
Fruit	1.436	-0.854
Edible oils and fats	0.941	-0.367
Sugar and gur	0.721	-0.729
Meats	1.350	-1.181
Vegetables	0.873	-0.456
Tea, coffee and soft drinks	1.075	-1.081
Milk and milk products	1.304	-0.993

Table 6 demonstrates the expenditure and own price elasticities for the upper class (*i.e.* the consumers belonging to 5th quintile). All the observed expenditure elasticities are of

Table: 6. Expenditure (Income) and Marshallian Own-price Elasticities for 5th Quintile

Food group	Expenditure	Own-price
Cereals	0.429	-0.470
Pulses	0.864	-0.231
Fruit	1.218	-0.636
Edible oils and fats	0.701	-0.127
Sugar and gur	0.607	-0.615
Meats	1.094	-0.925
Vegetables	0.653	-0.181
Tea, coffee and soft drinks	0.591	-0.597
Milk and milk products	1.114	-0.803

reasonable magnitude. The magnitude of the expenditure elasticities for this upper class, as per theoretical consideration and prior assumption, is low as compared to the poor class. Three groups reflect the tendency of being the luxury items like fruits, meats, and milk and milk products with expenditure elasticities 1.218, 1.094, and 1.114 respectively. No group reveals the status of Giffen commodity.

Cereal group for both of the income classes shows a behavior of basic need for the people. The expenditure elasticity of this group is lower as compared to all other included groups in both of the cases. It is overall 0.541, and 0.653 for 1st quintile and 0.429 for 5th quintile. As a basic need cereal group is less elastic towards the change in income as it has a certain fixed proportion in the expenditure of the households.

5.2. Uncompensated own-price elasticities

Uncompensated own-price elasticities of demand for all food groups are negative and consistent with the a priori expectation. The absolute amounts of these elasticities for all food groups are lower than unity except for meats in total sample of households as displayed in Table 4. The demand reacts in-elastically to own price changes. An exception is meat where the elasticity amounts to -1.053 (elastic) thus price changes affect the demand for meat in a greater extent as compared to the other included groups.

The uncompensated own-price elasticities for most the selected food groups, such as pulses, edible oils and fats, and vegetables are much lower than the total expenditure elasticities, implying that responsiveness of demand to own price changes of these aggregates is much lower than to variations in total expenditure. The largest absolute value of uncompensated own-price elasticity is calculated for the meat's group (*i.e.* -1.053). This implies that demand reacts elastically to changes in the prices of these products. The own price elasticities are lowest for pulses (-0.238), edible oils and fats (-0.247), and cereals (-0.582) where demand reacts least to price changes.

Having a look on Table 5, it is observed that meats; tea, coffee and soft drinks, and milk groups showed a high elastic attitude towards the change in own price, having own price elasticities -1.181, -1.081 and -0.993 respectively. While, on the other hand, pulses and edible oil groups depict a low magnitude of own price elasticities in absolute terms *i.e.* -0.245 and -0.367, respectively.

Table 6 reveals the information about the uncompensated own price elasticities for the rich class (5th quintile). No own price elasticity is found here, which have a magnitude greater than one in absolute terms. However, meat, and milk and milk products groups, with elasticities -0.925 and -0.803, respectively, reflect highly responsive towards the change in own price as compared to the other items pertaining to this aggregate food groups. On the

other side edible oil and fats, and pulses showed a very in-elastic behavior with elasticity magnitudes -0.127 and -0.231, respectively.

5.3. Compensated own-price elasticities

As predicted by demand theory, the compensated own-price elasticities are negative for all commodities (see table 8). For all commodity groups, they are lower in absolute terms than the uncompensated ones. Especially for vegetables, meats, and milk and milk product group, the compensated own-price elasticities are much smaller in absolute terms than the uncompensated ones, suggesting that a rise or fall in the price of the respective commodities would have considerable real expenditure effects.

5.4. Cross-price elasticities

The values of the cross-price elasticities are smaller - in absolute terms - than those of the expenditure or own-price elasticities. This holds true for uncompensated and compensated cross-price elasticities (see, Tables 7 and 8). The cross-price elasticities characterize pairs of goods as substitutes or complements. On the level of all selected food commodity groups, there are only substitution relationships and no complementary ones. As a matter of fact, in Pakistan, many diets are based on a single food with small amounts from plant or animal products. They lack dietary diversity. The fact that all food groups showed a substitution relation⁴ may be one reason explaining the lack of diversity in the Pakistani's diet. It is important that a number of different food sources be consumed and efforts should be made to encourage a wide variety of foods to improve the nutritional quality of the Pakistani's diet and health of the population. Dietary diversity is one of the most important ways to ensure a balance of nutrients for people of all ages. However, one would have expected a complementary relationship for cereal products with vegetable products, where in Pakistan, cereal products are frequently consumed jointly with vegetables (especially potatoes). This might result from aggregation decisions of the composite commodities.

5.5. Results by consumption quintiles

The LAAIDS model permits the calculation of elasticities for different consumption quintiles groups and HIES data materialized this happening. In order to do so, income and price elasticities for two extreme quintiles (1st and 5th) are estimated. It is obvious from table 5 to 6 and table 9 to 12 that poor

⁴ In order to observe the cross price relationships among the food items, a more detailed breakup of each food group (up to the individual commodity level) is needed.

people belonging to quintile 1st exhibit higher income elasticities for fruits, meats, milk and soft drinks groups as compared to the higher income groups (e.g. households belonging to 5th quintile). In other words, an increase in income of poor households will lead to higher expenditure on these commodity groups.

Table: 7. Uncompensated (Marshallian) Price Elasticities⁵ for Total Sample

Group ⁶	CR	PL	FR	EOF	SG	MT	VG	TCS	MMP
CR	-0.582	0.396	0.363	0.366	0.376	0.529	0.384	0.369	0.419
PL	0.768	-0.238	0.753	0.754	0.757	0.799	0.759	0.755	0.768
FR	0.279	0.295	-0.745	0.316	0.309	0.200	0.303	0.314	0.281
EOF	0.773	0.764	0.751	-0.247	0.757	0.816	0.760	0.754	0.772
SG	0.359	0.343	0.319	0.321	-0.672	0.441	0.334	0.323	0.357
MT	0.001	0.011	0.027	0.025	0.020	-1.053	0.017	0.024	0.001
VG	0.114	0.111	0.108	0.108	0.109	0.127	-0.290	0.108	0.114
TCS	0.179	0.171	0.159	0.160	0.164	0.219	0.167	-0.839	0.178
MMP	0.100	0.111	0.126	0.124	0.120	0.050	0.116	0.123	-0.898

Table: 8. Compensated (Hicksian) Price Elasticities⁷ for Total Sample

Group	CR	PL	FR	EOF	SG	MT	VG	TCS	MMP
CR	-0.502	0.449	0.377	0.385	0.406	0.739	0.423	0.390	0.492
PL	0.897	-0.153	0.777	0.786	0.847	1.097	0.817	0.802	0.891
FR	0.474	0.425	-0.710	0.361	0.382	0.715	0.399	0.366	0.368
EOF	0.894	0.845	0.773	-0.219	0.802	1.135	0.819	0.786	0.888
SG	0.456	0.407	0.336	0.344	-0.695	0.695	0.382	0.349	0.450
MT	0.180	0.131	0.059	0.067	0.088	-0.579	0.105	0.072	0.174
VG	0.165	0.145	0.116	0.120	0.128	0.261	-0.265	0.122	0.162
TCS	0.302	0.253	0.181	0.189	0.210	0.544	0.227	-0.806	0.296
MMP	0.278	0.229	0.157	0.165	0.186	0.519	0.203	0.170	-0.728

⁵ Uncompensated (Marshallian) own-price elasticities are written in bold letters.

⁶ cereals (CR), pulses (PL), fruits (FR), edible oils and fats (EOF), sugar and gur (SG), meats (MT), vegetables (VG), tea, coffee and soft drinks (TCS), and milk and milk products (MMP).

⁷ Compensated (Marshallian) own-price elasticities are written in bold letters.

Table: 9. Uncompensated (Marshallian) Price Elasticities⁸ for 1st Quintile

Group ⁹	CR	PL	FR	EOF	SG	MT	VG	TCS	MMP
CR	-0.694	0.284	0.251	0.254	0.264	0.417	0.272	0.257	0.307
PL	0.761	-0.245	0.746	0.747	0.750	0.792	0.752	0.748	0.761
FR	0.170	0.186	-0.854	0.207	0.200	0.091	0.194	0.205	0.172
EOF	0.653	0.644	0.631	-0.367	0.637	0.696	0.640	0.634	0.652
SG	0.302	0.286	0.262	0.264	-0.729	0.384	0.277	0.266	0.300
MT	-0.127	-0.117	-0.101	-0.103	-0.108	-1.181	-0.111	-0.104	-0.127
VG	0.550	-0.456	0.535	0.536	0.539	0.581	0.541	0.537	0.550
TCS	-0.063	-0.071	-0.083	-0.082	-0.078	-0.023	-0.075	-1.081	-0.064
MMP	0.005	0.016	0.031	0.029	0.025	-0.045	0.021	0.028	-0.993

Table: 10. Compensated (Hicksian) Price Elasticities¹⁰ for 1st Quintile

Group	CR	PL	FR	EOF	SG	MT	VG	TCS	MMP
CR	-0.614	0.337	0.265	0.273	0.294	0.627	0.311	0.278	0.380
PL	0.890	-0.160	0.770	0.779	0.840	1.090	0.810	0.795	0.884
FR	0.365	0.316	-0.819	0.252	0.273	0.606	0.290	0.257	0.259
EOF	0.774	0.725	0.653	-0.339	0.682	1.015	0.699	0.666	0.768
SG	0.399	0.350	0.279	0.287	-0.752	0.638	0.325	0.292	0.393
MT	0.052	0.003	-0.069	-0.061	-0.040	-0.707	-0.023	-0.056	0.046
VG	0.679	-0.371	0.559	0.568	0.629	0.879	0.599	0.584	0.673
TCS	0.060	0.011	-0.061	-0.053	-0.032	0.302	-0.015	-1.048	0.054
MMP	0.183	0.134	0.062	0.070	0.091	0.424	0.108	0.075	-0.823

⁸ Uncompensated (Marshallian) own-price elasticities are written in bold letters.

⁹ cereals (CR), pulses (PL), fruits (FR), edible oils and fats (EOF), sugar and gur (SG), meats (MT), vegetables (VG), tea, coffee and soft drinks (TCS), and milk and milk products (MMP).

¹⁰ Compensated (Marshallian) own-price elasticities are written in bold letters.

Table: 11. Uncompensated (Marshallian) Price Elasticities¹¹ for 5th Quintile

Group ¹²	CR	PL	FR	EOF	SG	MT	VG	TCS	MMP
CR	-0.470	0.508	0.475	0.478	0.488	0.641	0.496	0.481	0.531
PL	0.775	-0.231	0.760	0.761	0.764	0.806	0.766	0.762	0.775
FR	0.388	0.404	-0.636	0.425	0.418	0.309	0.412	0.423	0.390
EOF	0.893	0.884	0.871	-0.127	0.877	0.936	0.880	0.874	0.892
SG	0.416	0.400	0.376	0.378	-0.615	0.498	0.391	0.380	0.414
MT	0.129	0.139	0.155	0.153	0.148	-0.925	0.145	0.152	0.129
VG	0.223	0.220	0.217	0.217	0.218	0.236	-0.181	0.217	0.223
TCS	0.421	0.413	0.401	0.402	0.406	0.461	0.409	-0.597	0.420
MMP	0.195	0.206	0.221	0.219	0.215	0.145	0.211	0.218	-0.803

Table: 12. Compensated (Hicksian) Price Elasticities¹³ for 5th Quintile

Group	CR	PL	FR	EOF	SG	MT	VG	TCS	MMP
CR	-0.390	0.561	0.489	0.497	0.518	0.851	0.535	0.502	0.604
PL	0.904	-0.146	0.784	0.793	0.854	1.104	0.824	0.809	0.898
FR	0.583	0.534	-0.601	0.470	0.491	0.824	0.508	0.475	0.477
EOF	1.014	0.965	0.893	-0.099	0.922	1.255	0.939	0.906	1.008
SG	0.513	0.464	0.393	0.401	-0.638	0.752	0.439	0.406	0.507
MT	0.308	0.259	0.187	0.195	0.216	-0.451	0.233	0.200	0.302
VG	0.274	0.254	0.225	0.229	0.237	0.370	-0.156	0.231	0.271
TCS	0.544	0.495	0.423	0.431	0.452	0.786	0.469	-0.564	0.538
MMP	0.373	0.324	0.252	0.260	0.281	0.614	0.298	0.265	-0.633

VI. Conclusion and Policy Recommendations

Lack of dietary diversity is a particular problem among the people in Pakistan, because their diets are predominantly based on starchy staples with little animal products and few fresh fruits and vegetables. It is observed that the major sources of calories and proteins in Pakistan are plant products with small amounts from animal products as a concentrated source of essential protein that are of high quality and highly digestible. In addition, the diets in Pakistan are low in fat intake, since of all basic foodstuffs, fat is one of the

¹¹ Uncompensated (Marshallian) own-price elasticities are written in bold letters.

¹² cereals (CR), pulses (PL), fruits (FR), edible oils and fats (EOF), sugar and gur (SG), meats (MT), vegetables (VG), tea, coffee and soft drinks (TCS), and milk and milk products (MMP).

¹³ Compensated (Marshallian) own-price elasticities are written in bold letters.

most expensive. Therefore, in Pakistan, the consumers are still suffering from malnutrition and unbalanced essential nutrients like caloric value, proteins, and fat content. Also, there is a marked difference between rural and urban areas in food consumption patterns.

It is explored that the expenditure and price elasticities for selected food groups are relatively high in Pakistan. As expected, the estimation results show that expenditure elasticities for all food groups are positive and less than one, except for fruits, meats, and milk; indicating that the selected food groups are necessities. For food groups such as fruits, meats, and milk having expenditure elasticities larger than unity, identifying them as luxuries, it is expected that these food groups will experience an increase in demand when consumers' income increases in tandem with the overall economic growth of the country.

Another interesting finding is that cereals tend to have the lowest expenditure elasticity of demand. This indicates that cereals have already occupied a special position in the Pakistan's diet, as it is the staple food of the population. Uncompensated own-price elasticities of demand for all food groups are negative and consistent with the theoretical expectation. The absolute amounts of these elasticities for all commodity groups are lower than unity and so the demand reacts in elastically to own price changes, except for meats amounting to -1.053 (elastic). The uncompensated own-price elasticities (in absolute value) for most food groups, such as pulses, oils and fats, and vegetables than the total expenditure elasticities, implying that food demand reacts more elastically to expenditure changes than to own price changes. The elasticities are lowest (in absolute value) for vegetables (-0.290), oils & fats (-0.247), and cereals (-0.582) where demand reacts least to price changes.

For all commodity groups, the compensated own-price elasticities are lower - in absolute terms - than the uncompensated ones, suggesting that a rise or fall in the price of the respective commodities would have considerable real expenditure effects. According to the values of cross-price elasticities and on the level of all selected food commodity groups, only substitution relationships are observed. Many diets in Pakistan are based on a single of food with small amounts from vegetables or animal products and lack dietary diversity in the diet, which supports this result. However, one would have expected a complementary relationship for cereal products with vegetables, because in Pakistan, cereal products are frequently consumed jointly with

vegetables (especially potatoes). This might result from aggregation decisions of the composite commodities.

The findings of the empirical analysis of price and expenditure (income) elasticities for the selected food groups could be used in the projections for future food consumption. Pakistan is expected to be getting farther and farther away from being self-sufficient in its food production. This holds true particularly for food items exhibiting high expenditure elasticities such as livestock products. The high price elasticities of demand for many food items stress the importance of food price changes for Pakistani households, and their reactions should be taken into account in the development of comprehensive agricultural and food policies in order to avoid unattended effects harming consumers.

Due to the strong influence of diets on health, adequate food consumption is an important public health concern. In Pakistan, diets are traditionally overly rich in calories due to high consumption of cereal products and comparatively low consumption of healthy food such as fruits and livestock products. It is important, therefore, that efforts undertaken to encourage consumption of a wide variety of foods to improve the nutritional quality of the diet and health of the population. Considering the relatively high expenditure elasticities of demand for fruits and livestock products of all households, income increases would exert a positive influence on the intake of micronutrients that are delivered by fruits and livestock products. The results of this study suggest that income oriented policies are important to achieve better nutrition and reduce the problem of unbalanced diets in Pakistan. In addition, complementing policies are necessary.

Since Pakistan has a high income inequality, it is expected that income and price-elasticities are different between the richest and the poorest. The results supported this expectation, indicating that income-elasticities are higher for the poorest for all staple food. Moreover, own-price elasticities are higher for the poorest households in the case of cereals and pulses, the most consumed staple food commodities in Pakistan. These results are an important step forward in understanding household consumption habits in Pakistan, and highlight the consumption differences between poor and rich in the country. The elasticities calculated in this study are powerful instruments in helping policymakers in devising policies targeted at poor people.

Food subsidies can be better targeted to the poor people by subsidizing food items and distributing in villages and rural neighborhoods where the poor

are known to be concentrated. The total annual food subsidy resources could be allocated to each region according to its contribution to total poverty. The subsidy system should re-establish subsidies on some of the healthy foods like red meat and fish because these items are a relatively concentrated source of essential protein of high quality and highly digestible. The best way for Pakistan to improve its food distribution system is that the food subsidy system should be changed from the commodities form to a cash subsidy provided only to low-income households and reduces the benefits to the non-needy.

Increase in animal production must be focused, particularly small ruminants and fisheries, aiming at increasing the per capita consumption of animal protein in its various forms by means of raising productivity of domestic cattle of buffalo, cow and sheep using improved genetic techniques; and by introducing high-yield genetics as a means to increase milking rate, meats and eggs production. Increasing the quantities of animal products is expected to have an effect on the prices as a whole and as a result may benefit consumers. Decrease per capita consumption of cereals through redistribution of flour uses, raising the standard of living of the population and changing food consumption patterns.

It is important that a number of different food sources be consumed and efforts should be made to encourage a wide variety of foods to improve the nutritional quality of the Pakistani's diet and health of the population. Dietary diversity is one of the most important ways to ensure a balance of nutrients for people of all ages. The results of this study suggest that income oriented policies are important to achieve better nutrition and reduce the problem of unbalanced diets in Pakistan.

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